

A Working Paper Draft

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"LOGARITHMIC CURVES"

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# "LOGARITHMIC CURVES"

## Abstract

The nature of logarithms and their use in plotting certain phenomena are discussed.

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## Why Logarithms?

Many phenomena in nature and society have properties in which the quantity changes exponentially with time. By taking the logarithm of these quantities, one can plot these curves on logarithmic scales as straight lines or almost straight lines. When using logarithms, the calculation of the interaction between different quantities is simplified, because multiplication and division of numbers becomes addition and subtraction of logarithm of numbers.

## What are Logarithms?

Definition: Logarithm: The exponent that indicates the power to which a number is raised to produce a given number. (The logarithm of 100 to the base 10 is 2).

Consider a living cell which divides into two cells every hour. These two cells then each divide into two each making four cells at the end of two hours as is illustrated in Figure 1.

The number of cells vs. time is shown in Figure 2. The "logarithm to the base 2" of a number is number of times 2 is multiplied by itself to get the number. i.e. The "logarithm to the base 2" of "8" is "3", because three twos multiplied by each other gives eight:  $2 \times 2 \times 2 = 8$ .

## How are Logarithms Useful?

If we want to know how many cells we will have at a future time say, 8 hours from the start we can draw a graph on logarithmic paper in which the curve of Fig. 2 will be a straight line as is shown in Fig. 3.

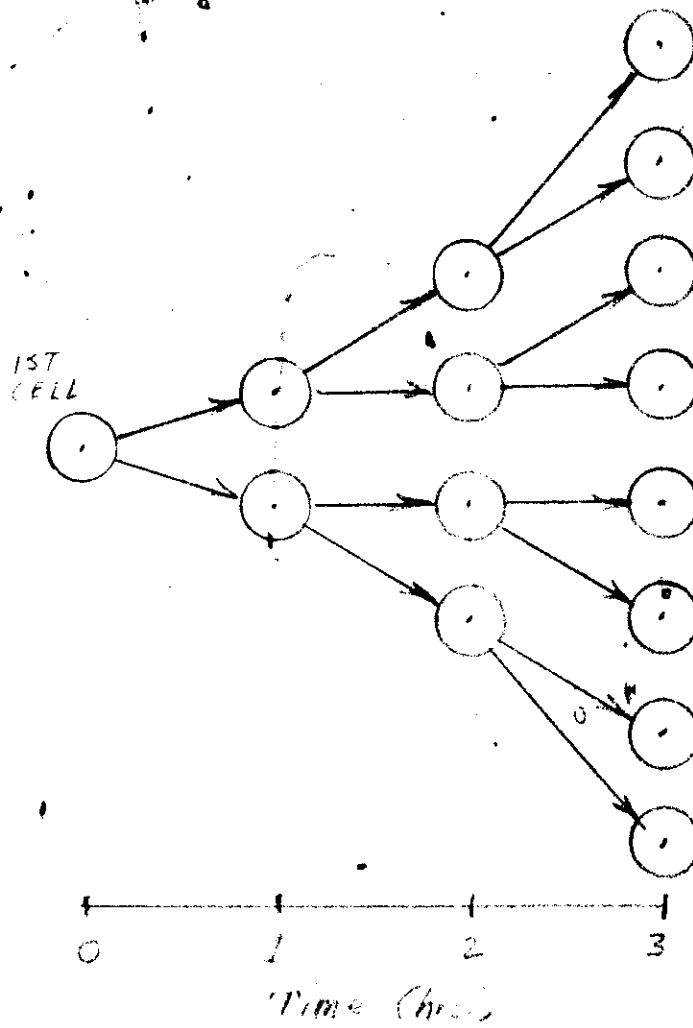


Fig. 1 - Division of Cells with Time.

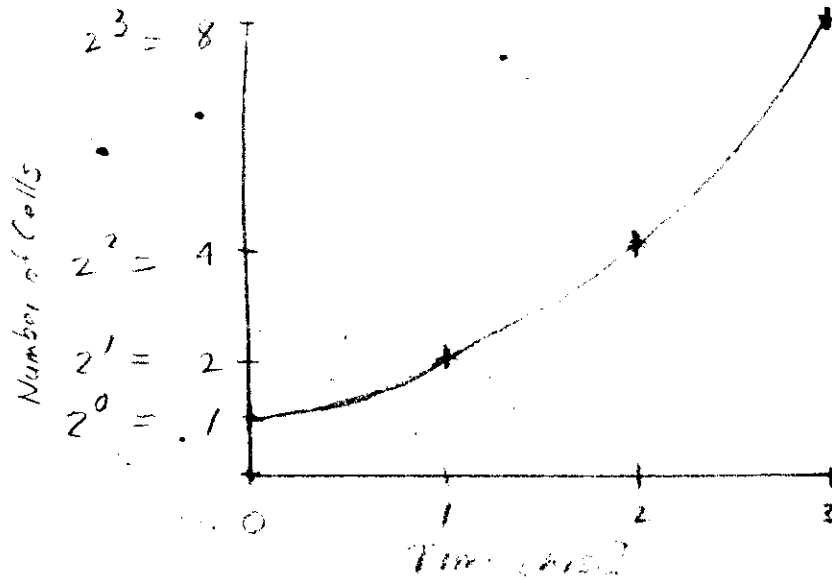


Fig. 2 - Number of Cells vs. Time

What Kinds of Logarithms are There?

In addition to "logarithms to the base 2" there are numerous other logarithms to different bases.

Common Logarithms:

The most common type used is the "common logarithm" or "logarithm to the base 10", where:

logarithm	1000 = 3	since	10x10x10 = 1000
logarithm	100 = 2		10x10 = 100
logarithm	10 = 1		10 = 10
logarithm	1 = 0		zero 10's in 1
logarithm	0.1 = -1		$\frac{1}{10} = 0.1$
logarithm	0.01 = -2		$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = .01$

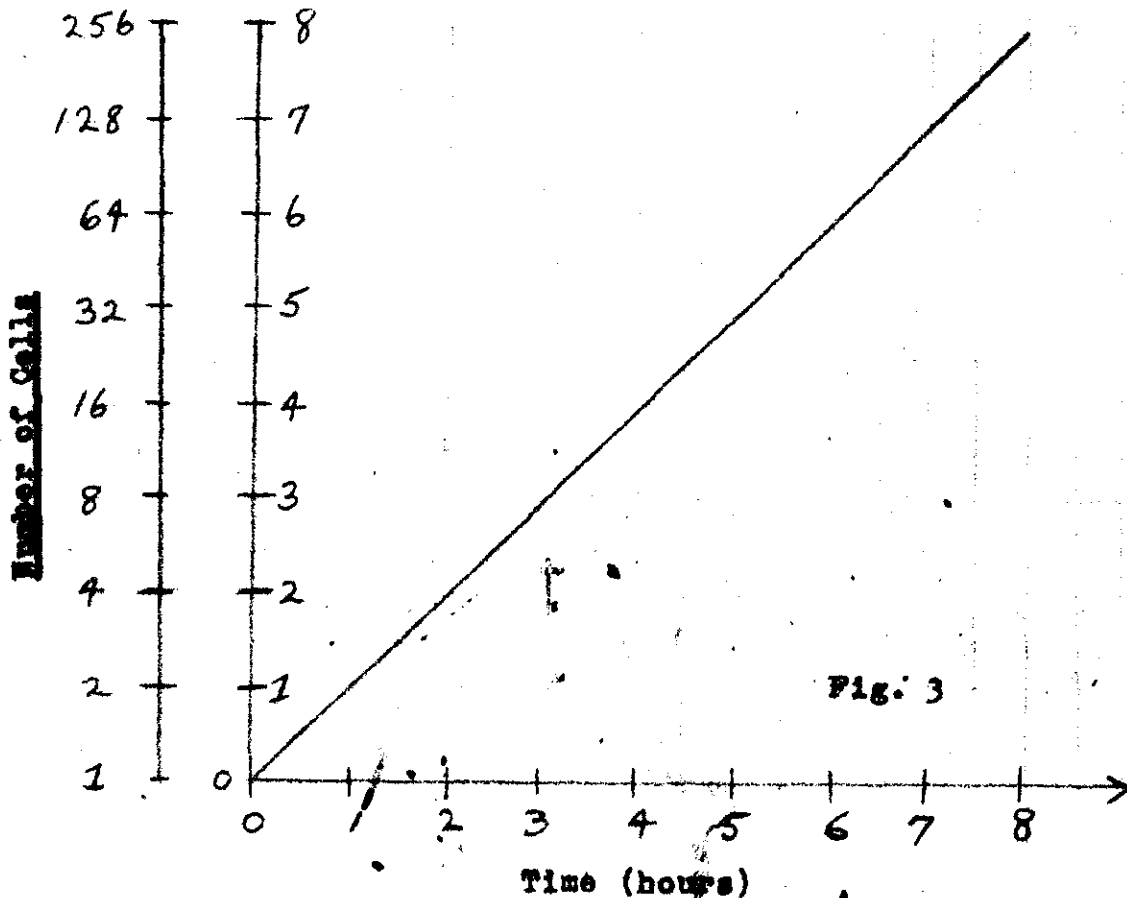


Fig. 3 - Number of Cells vs. Time on Semi-Log Paper.

Common logarithms are used for convenience, because one can multiply numbers by adding their logarithms. This is the principle that make possible the slide rule.

### Natural Logarithms

Many phenomena in nature, particularly physics, either grow or decay by the formula:

$$(2.718282)^x \text{ or } (2.718282)^{-x}$$

where the above formula means that 2.718282 or  $\frac{1}{2.718282}$  is multiplied by itself x times.

The number 2.718282 is called "e".

"log to the base e" of 7.39 is 2, because

$$2.718282 \times 2.718282 = 7.38906 \dots\dots\dots$$

### Philosophical Significance of Logarithmic Growth Curves

The different epochs of the development of our civilization are plotted in Fig. 4. With the exception of a few gaps the major steps in the development of human life follow a uniform distribution on this semi-logarithmic graph. Other items are yet to be inserted which may fill in the gaps.

Above this chart are marked the levels of the "Manifest Conscious" and the "Latent Unconscious" of the human brain as described by Foulkes and Anthony.\*

Although the scales are not coincident, I believe that

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\*S.H. Foulkes and E.J. Anthony Group Psychotherapy, The Psycho-Analytic Approach Penguin Books; Pelican Psychology Series A 370. (1957) p. 246 in Chapter 9: Metatheory: Speculations on Theoretical and Practical Developments.

this organization of the material will make more plausible to the physical scientist the process of the development (or evolution) of the "unconscious" and "conscious" components of the brain postulated in psycho-analytic theory.

Notes in the margin (not very legible) indicate additional stages to be added to this chart. An implimenting force in the development of this chart is the development of information theory. The concept of negative feedback circuits is expected to be very useful in developing a model of the maturation of an individual.

An erroneous conclusion might be inferred from Fig. 4 that progress goes "upward and onward forever" at this logarithmic rate. There must be superimposed a cyclical filter due to the cyclical variation of the distance of the earth from the sun which caused the alternating glacial epochs. This produces a band-limited function analogous to the filters used in telephone and telegraph circuits.

This growth curve is also dependent upon an optimization of the "negentropy" if we may borrow from information theory and thermodynamics.

This note was intended to be an explanation for the layman of the simple use of logarithmic curves, but has developed into the "kernel" of a view of the relationship of our minds to the evolution of mankind.

Albert Schweitzer's motto "Reverence for Life" may sometime be symbolized by:

$$H = - \sum_1 P_i \log P_i$$

September 18, 1957

F.B. Wood

Numbers refer to items  
in Fig 2 of memo. of 4-18-56

Sept 17, 1957  
F. B. Wood

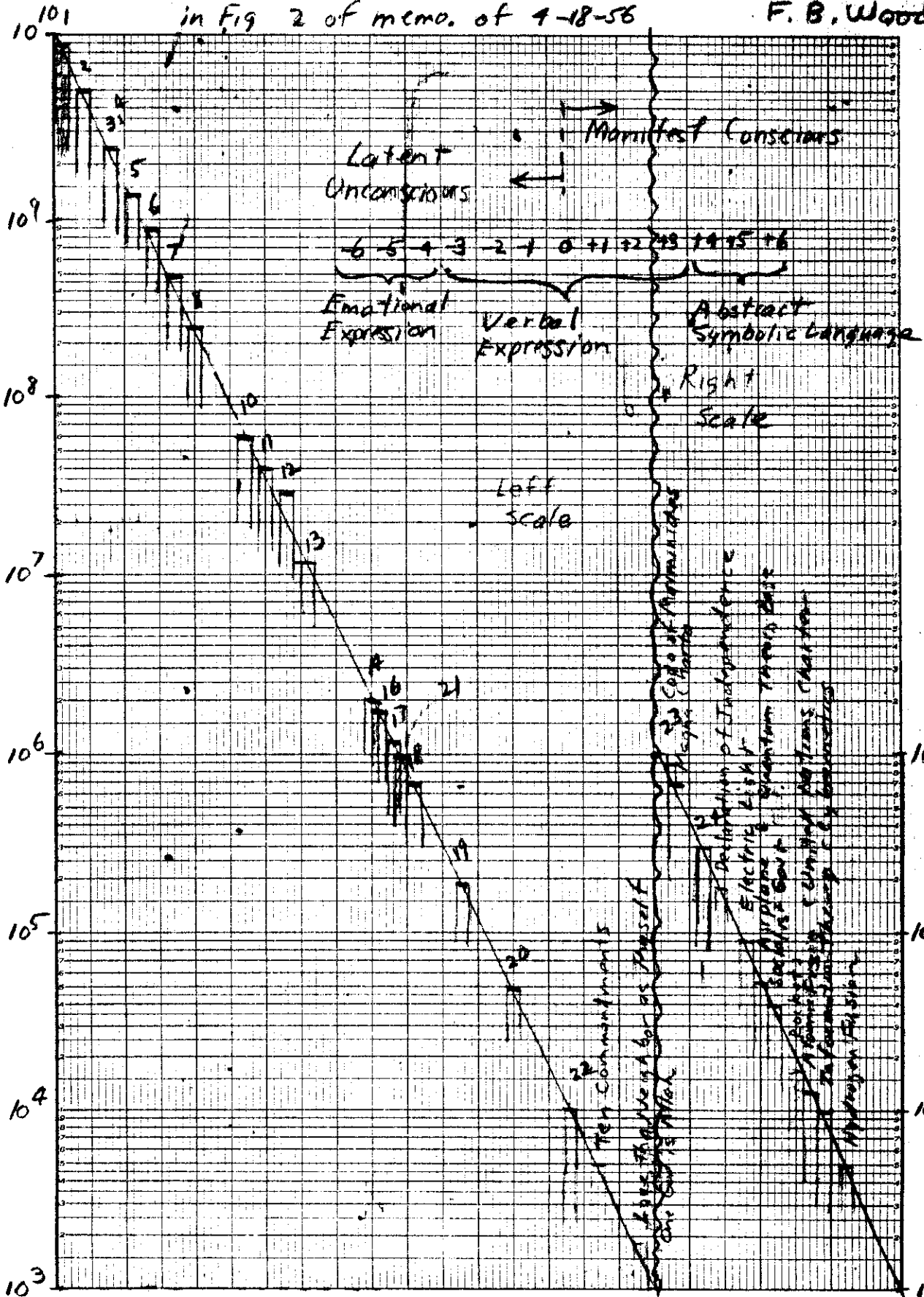


Fig. 4



## Appendix A

The following figures give more detail for the curve of Fig. 4.

Fig. A-1 is a picture of our universe.

(This is the basis of Fig. 6 in SEPR No. 18-B)

Figs. A-2a & A-2b give more details for Fig. 4.

The numbers in these figures correspond to those in Fig. 4.

### References and Sources:

Figure A-1 - Copyright by Simon and Schuster

Figure A-2 -

- Ref. 4. Edwin Hubble, "The problem of the expanding universe," Science in Progress, Third series, Yale Univ. Press, pp. 22-44, 1942.
5. Pascual Jordan, "Die physik des 20. jahrhunderts," 1936, trans. by Eleanor Osry, p. 29, 1944.
6. Ibid., p. 176
7. John Phelan, "Anthropological backgrounds," in Elmer Pendell (ed.), Society under analysis, An introduction to Sociology, p. 9, 1942.
8. Ibid., pp16-17.
9. Selden Snyser, "Logics: Subverbal, and superverbal," in Papers from the First American Congress for general semantics, March 1935, Arrow editions, N.Y., 1940.

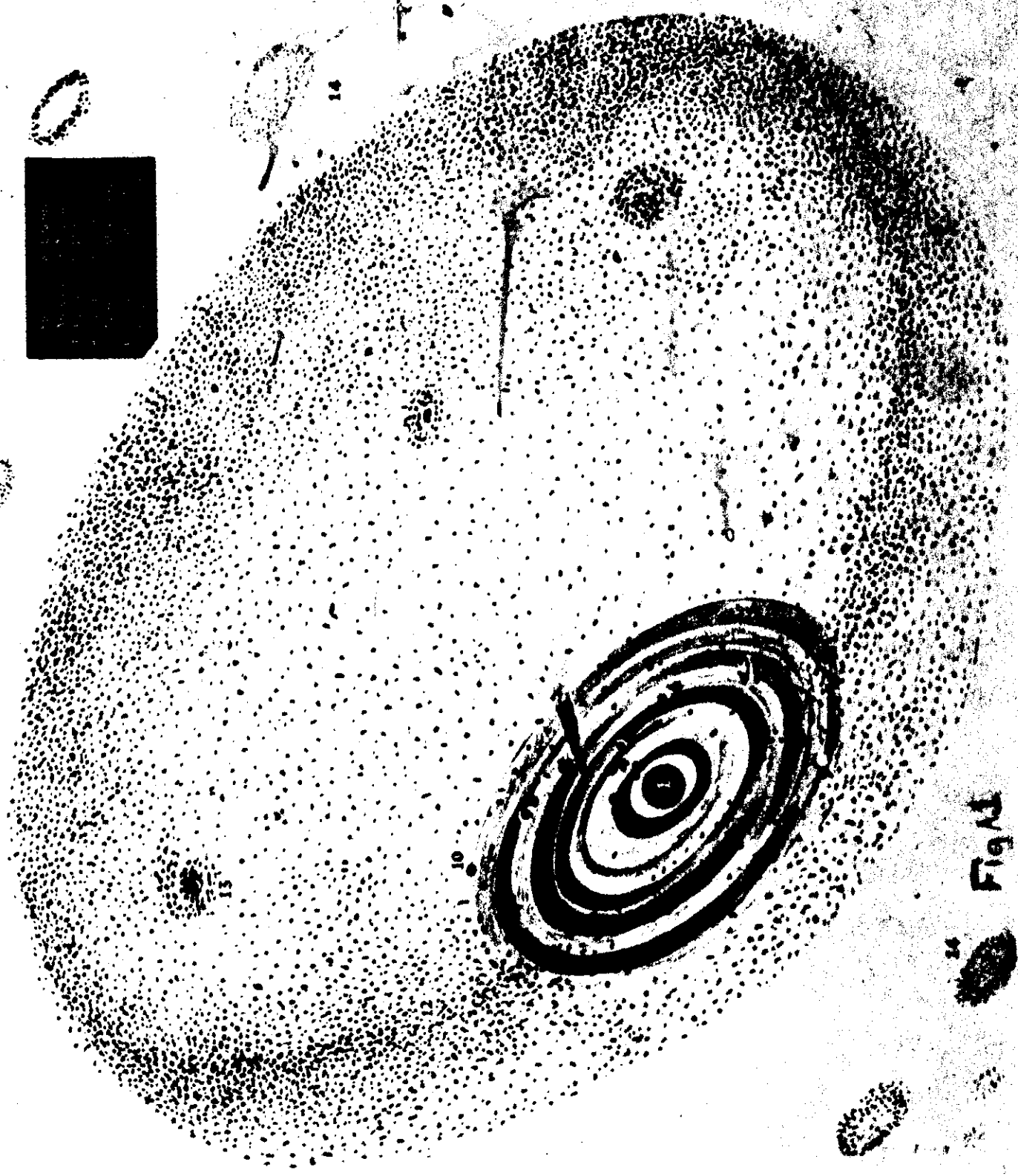
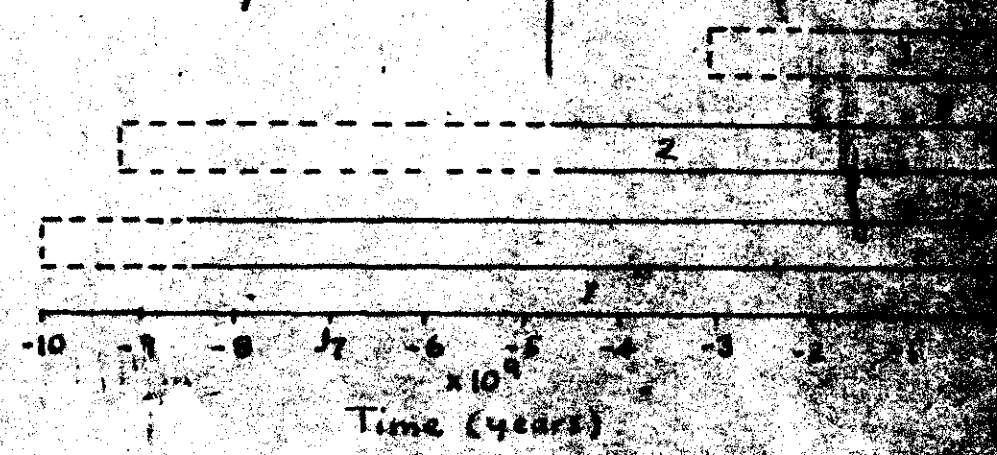
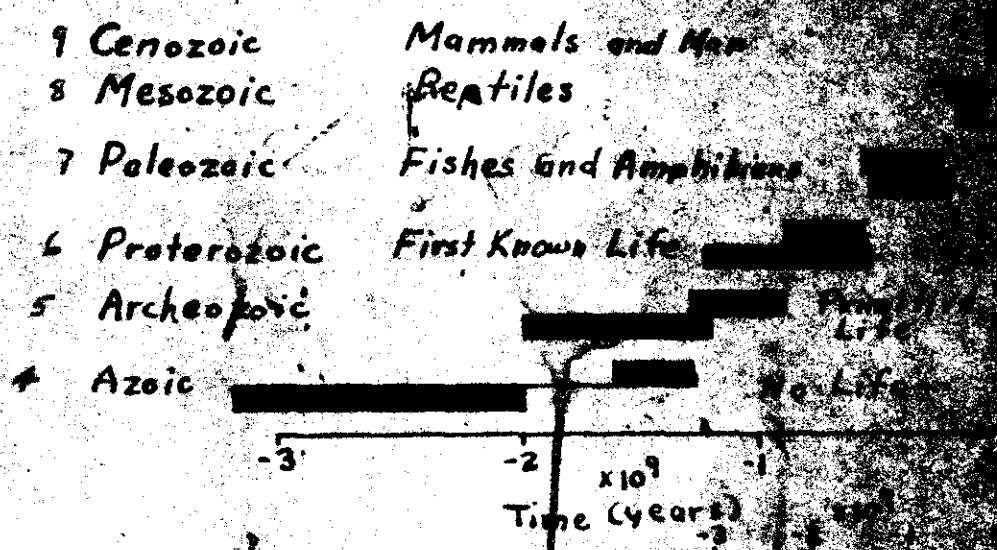
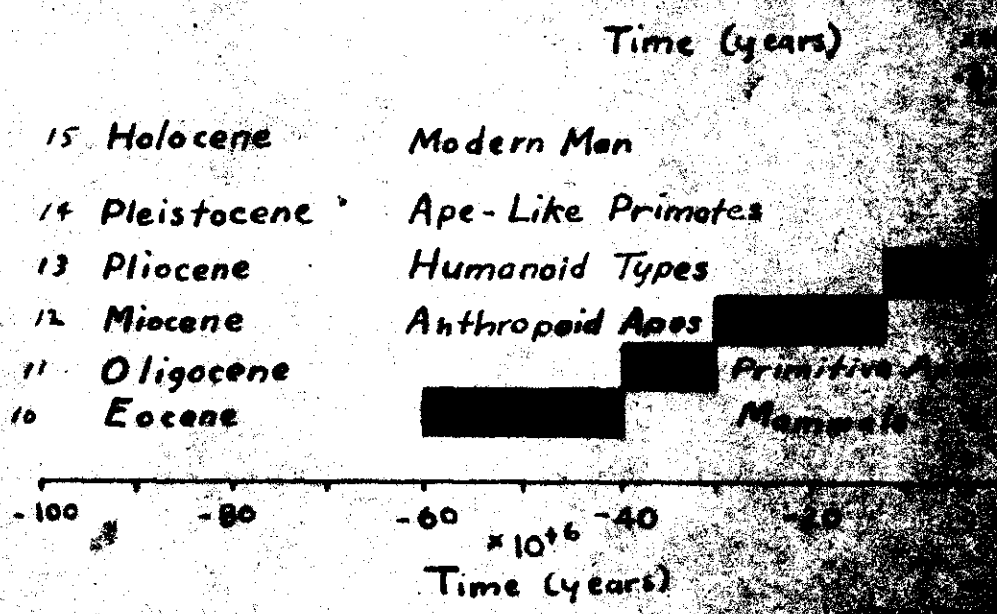


Fig 1

The following comments are my own, and are offered to supplement the idea of logarithmic growth.

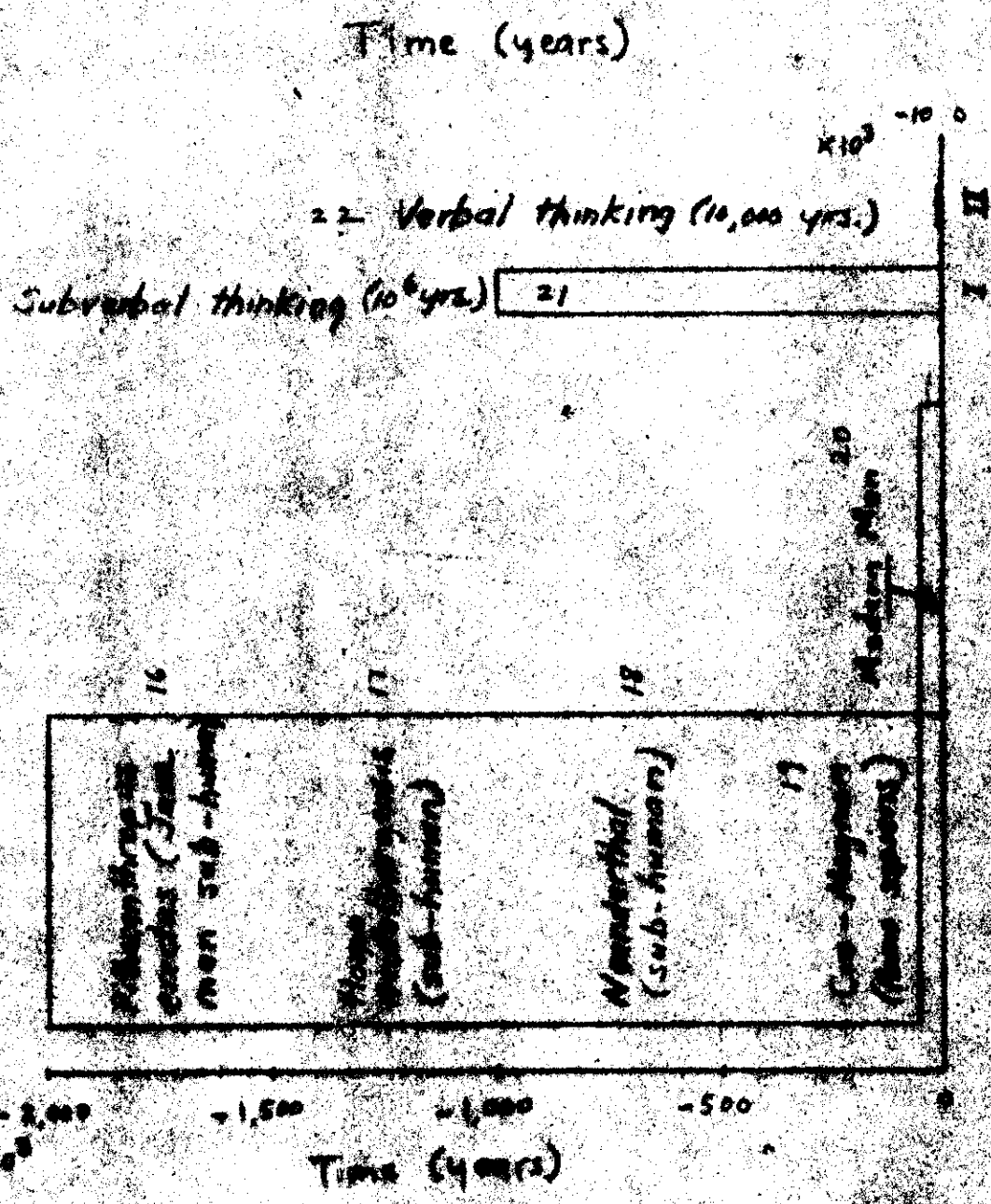
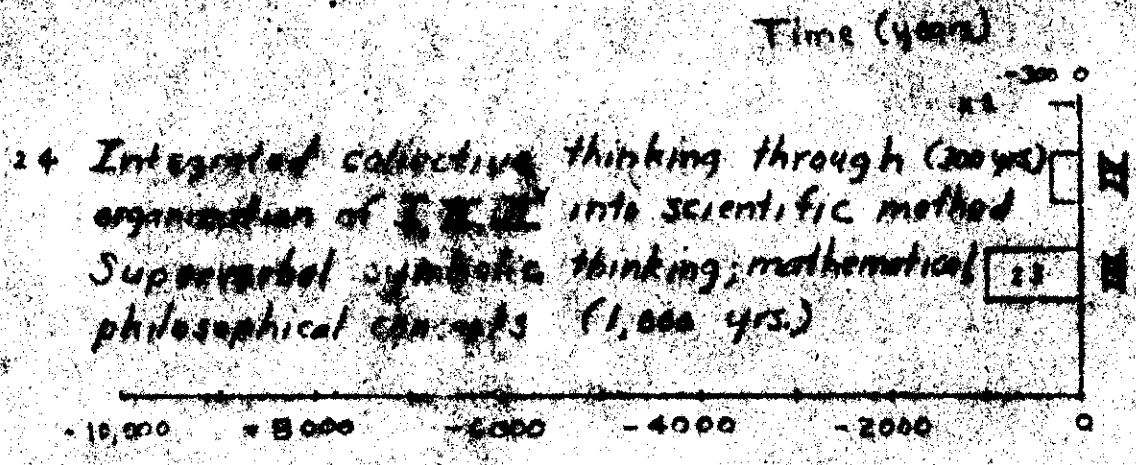
First I wish to show by illustration some features of growth in our universe. Consider the wondrous universe of which we are a part (Fig. A-1). The graphs in Fig. A-2 show the relative age of the universe, our solar system, and the earth. Preceding from the bottom of the page up, the second part is an expansion of the age of the earth into the different geological eras. The cenozoic age during which the mammals and finally man developed is expanded in the top section. If one compares the time of development of each successive evolutionary step, one can deduce a trend shown by the curved lines. This trend shown by the curved lines indicates that the time of development for each higher or more specialized step is shorter than the previous step.

Further expansion of the development of man and the evolution of man's thinking are shown in the second page of Fig. A-2. When one plots the production of energy in terms of tons of coal or electric energy one has numerical values that can be plotted. Instead of plotting such curves, I am giving some average values. In 1897 the rate of increase of total power production for the whole world was such that the total energy produced per year would be doubled every fifteen years. Except for discontinuities caused by wars and depressions the world rate has continued to be the same. Electrical energy production in the U.S.A. from 1915 to



# HISTORICAL PERSPECTIVE

Fig. A2.4



Evolution of Man's Thinking

Development of Man

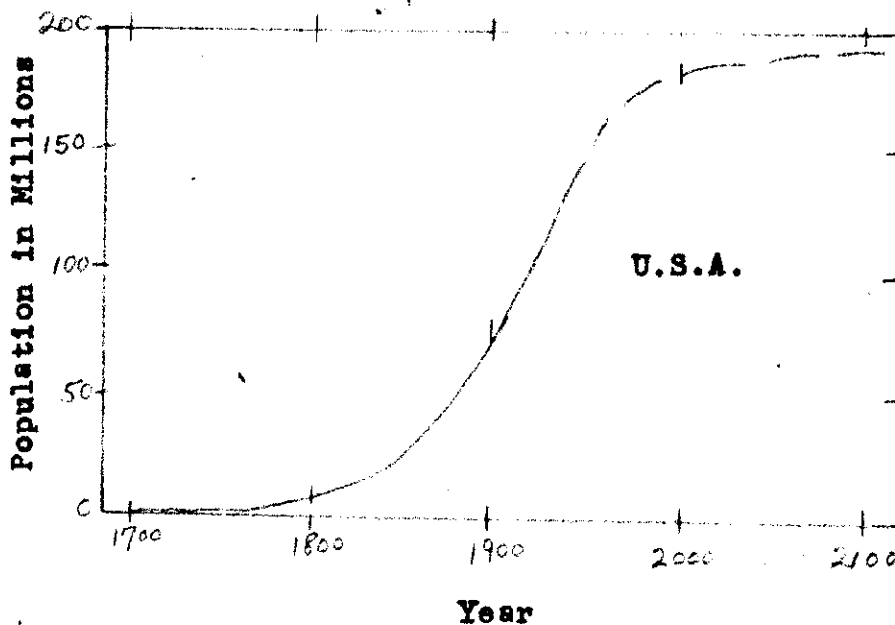
Generalized Historical Perspective (Part Two)

1947 has averaged around a ten year doubling period.

The doubling period for any activity is determined by plotting data on semi-logarithmic graph paper.

Note on Limitation of Logarithmic Increase.

Morris Kline\* has pointed out that the logarithmic increase of population is a geometrical progression like Malthus asserted. Kline gives an equation on curves developed by Pearl and Reed in which population growth curves taper off toward a saturation value.



$$Y = \frac{L}{1 + a(2.718)^{kt}}$$

$$Y = \frac{197.27}{1 + 67.32(2.718)^{-.0313t}} \text{ Millions}$$

t = years since 1780

\* Morris Kline Mathematics in Western Culture, N.Y.: Oxford (1953) p. 337.