

2.0 Signal Space ( $\Sigma$ ).

$$(\Sigma) = (S_1, \dots, S_i, \dots, S_m)$$

signal vector

$$S_i = [a_{i1}, a_{i2}, \dots, a_{ik}]$$

## 2.1 Bit Signal Space

Considering the signal space  $\Sigma$  for a business data transmission code such as the Transcode or 4-out-of-8 code (see p 2-2), leads to the subdivision of the signal space by bit and by characters. For example, the letter "A" is represented as:

$$S_A = S_A(t) = S(t) \begin{bmatrix} a_{A1} \\ a_{A2} \\ a_{A4} \\ a_{A7} \\ a_{A8} \\ a_{A9} \\ a_{Ax} \end{bmatrix} = S(t) \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad (2.1)$$

bit signal

width

The "bit signal" may be of numerous forms:

$$S^c(t) = \cos \frac{2\pi t}{T} \quad \text{for } 0 \leq t \leq T \quad (2.2)$$

$$S^{hc}(t) = \cos \frac{\pi t}{T} \quad \text{for } 0 \leq t \leq T \quad (2.3)$$

$$S^s(t) = \sin \frac{2\pi t}{T} \quad \text{for } 0 \leq t \leq T \quad (2.4)$$

Note that the extension of Fig 1-2 to the transmission line case requires a modification of "⊗" to account for the delay characteristics.



The probability density function has two components: (1) bit probability and (2) character probability. The bit probabilities for the 4-out-of-8 code are:

$$\sigma(a_{ij} = 1) = 0.5$$

$$\sigma(a_{ij} = 0) = 0.5$$

## 2.2 Character Space $\Omega$

Using the 4-out-of-8 code as an example, the probability distribution functions are:

Alphabetic  $\sigma_A'(s_i)$  where  $\sum_{i=1}^{26} \sigma_A'(s_i) = 1.0$   
 determined from tables of usage of English letters in writing.

Numeric:  $\sigma_N'(s_j)$  where  $\sum_{j=0}^9 \sigma_N'(s_j) = 1.0$   
 Unless facts show some weighting exists, assume equal probability for each numeric.

$$\sigma_N'(s_j) = 0.1$$

Control Code.  $\sigma'_c(S_h)$  - where  $\sum_{h=1}^{18} \sigma'_c(S_h) = 1.0$   
(also special symbols)

As a first approximation the proportion of usage of special symbols + control code symbols can be estimated from present transceiver operations.

Unused (Reserve) Symbols.

$$\sigma'_u(S_e) \quad \text{where} \quad \sum_{e=1}^{16} \sigma'_u(S_e) = 1.0$$

where the probabilities of use of classes of symbols are defined as follows:

$\alpha_A$  = Probability of alphabetic symbol occurring

$\alpha_N$  = " " numeric " "

$\alpha_c$  = " " control code or special symbol occurring

$\alpha_U$  = " " unused or reserve symbol occurring

Then the character probabilities are related by:

$$\alpha_A \sum_{i=1}^{26} \sigma'_A(S_i) + \alpha_N \sum_{j=0}^9 \sigma'_N(S_j) + \alpha_c \sum_{h=1}^{18} \sigma'_c(S_h) + \alpha_U \sum_{e=1}^{16} \sigma'_u(S_e) = 1.0$$

Alternatively let  $\sigma_m = (\alpha_A \sigma'_A(S_1), \alpha_A \sigma'_A(S_2), \dots, \alpha_U \sigma'_u(S_{16}))$

then  $\sum_{m=1}^{70} \sigma_m = 1.0$

Question: Can these signal probability functions be profitably put in matrix form?

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Information

¶68 §9.3 The Information Channel as a Matrix  
Noiseless channel.

$$\widehat{CH} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

p70

$$\vec{P} \cdot \widehat{CH} = \vec{Q}$$

$$\left[ \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{8} \right] \cdot \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & 0 & 0 \end{bmatrix} = \left[ \frac{5}{32} \quad \frac{11}{32} \quad \frac{10}{32} \quad \frac{6}{32} \right]$$

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5-29-57

Question: Can a three dimensional  
probability matrix be developed?  
J.E. can the bit probabilities and  
character coding be introduced in the  
third dimension?

FBW