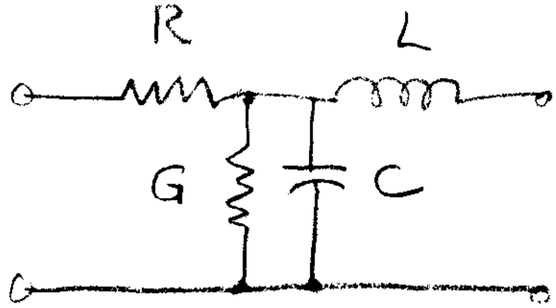
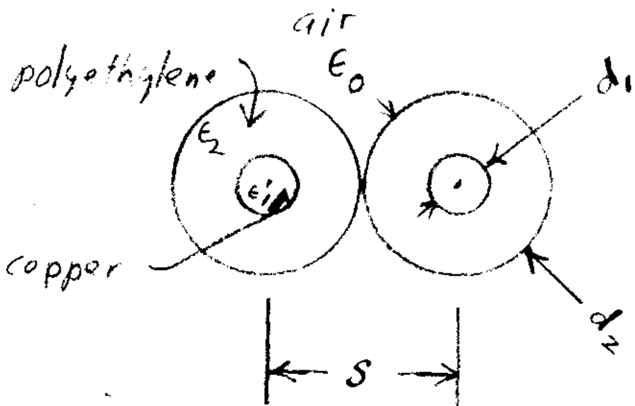
11.11 PRIMARY CABLE CONSTANTS (R, L, G, C)

Refer to the following report:

F. B. Wood, "Survey of the Formulas for Primary Cable Constants", San Jose: IBM Research Laboratory. Report RJ-134. Oct. 21, 1958.

11.11 Primary Cable Constants (R, L, G, C)





$$r_1 = \frac{d_1}{2}$$

Resistance

Proximity Effect

$$R \text{ (ohms/meter)} = \frac{2R_s}{\pi d_1}\left[\frac{s/d_1}{\sqrt{(s/d_1)^2-1}}\right] \qquad (1)^{a}$$

MKS
VHF

$$R = \left[\frac{2R_s}{\pi d_1}\right]\left[\sqrt{2}\,\frac{\text{Ber}\,q\,\text{Bei}'q - \text{Bei}\,q\,\text{Ber}'q}{(\text{Ber}'q)^2 + (\text{Bei}'q)^2}\right] \times$$

$$\times \left[\frac{s/d_1}{\sqrt{(s/d_1)^2-1}}\right] \qquad (2)^{b}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma}} \text{ ohms} \qquad (3)$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{0.0660}{\sqrt{f}} \text{ meters (for copper)} \qquad (4)$$

$$q = \frac{\sqrt{2}\, r_1}{\delta} \qquad (5)$$

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a. Ramo & Whinnery, Fields and Waves in Modern Radio (1st Ed) Table 9.01

b. Ref a, p 213 eq (7) multiplied by 2 for two wires and by proximity effect.

The Ber β and Bei β function are defined by the zero order Bessel function:

$$J_0(j^{-\frac{1}{2}} v) \equiv Ber(v) + j\ Bei(v) \quad (6)$$

For $\beta \to \infty$

$$R = \frac{2R_s}{\pi d_1}\left[\frac{s/d_1}{\sqrt{(s/d_1)^2 - 1}}\right] \quad (7)$$

Curves are available[c] for obtaining the second term in ξ_0 (2).

For low frequencies:

$$R = R_0\left[1 + \frac{1}{48}\left(\frac{r_1}{\delta}\right)^4\right] \quad (8)$$

For D-C:

$$R_0 = \frac{2}{\pi r_2^2 \sigma} \text{ ohms / meter} \quad (9)$$

The factor 2 in the numerator is for the two conductors.

The proximity effect was derived for the high frequency case. Sample experimental curves for lower frequencies, are available[ca] See also the sources on proximity effect.[cb]

c. Ref a pp 213-214. Fig 6.09a, 6.09b.

ca. Bell System Practices.

cb. RE Terman Radio Engineers Handbook (1st Ed) p36
H B Dwight "Proximity Effect in Wires and Thin Tubes" Trans AIEE, Vol 42, p 850 (1923)

Inductance

Inductance of two wire in a loop consists of two parts; the external inductance L_o due to flux linkages external to the wires and the internal inductance L_i due to flux linkages inside the conductors.

At d-c: (11-10)

$$L_o = \frac{\mu}{\pi} \cosh^{-1}\left(\frac{S}{d_1}\right) \quad \text{OK} \qquad (10)^{d,}$$

$$L_i = \mu \qquad \text{Divide by } 8 \;(?) \qquad (11)$$

For a general discussion of line parameters refer to Guillemin.[f] He derives an equation for L for the dissipationless case:[g]

$$L_{total} = 4\mu \ln\left(\frac{a + \sqrt{a^2 - r_1^2}}{r_1}\right), \qquad (12)$$

$$a = \frac{S}{2}, \qquad (13)$$

and a zero frequency dissipative case:

$$L_o + L_i = 4\mu \ln\left(\frac{2a - r_1}{r_1}\right) + \mu \quad \text{units are wrong} \qquad (14)$$

(ext.) (int)

Eq (14) approaches (12) at very high frequencies for the dissipative case.

d. Ref a Table 9.01, also c:
e. Smythe, Static and Dynamic Electricity (2nd Ed.) p 462, eq (4
f. E.A. Guillemin Communication Networks vol II (1935) pp 24-27.
g. Ref f pp 23, 25.

Eq (10) and (12)(14) should be compared for consistency when used in report.

Is the internal term in Eq (14) for bothwires?[k] This can be checked by sample calculation for #19 gauge wire in addition to following derivations in text and in Woodruff[h]

The high frequency inductance is in terms of the resistance component due to internal inductance:

$$\omega L_i = 2\left[\frac{R_s}{\pi d_1}\right]\left[\sqrt{2}\,\frac{\text{Ber}\,q\,\text{Bei}'q + \text{Bei}\,q\,\text{Ber}'q}{(\text{Ber}'q)^2 + (\text{Bei}'q)^2}\right]\left[\frac{s/d_1}{\sqrt{(s/d_1)^2-1}}\right] \quad (1$$

The high frequency limit is:

$$\omega L_i = 2\left[\frac{R_s}{\pi d_1}\right]\left[\frac{s/d_1}{\sqrt{(s/d_1)^2-1}}\right] \quad (16)$$

Curves of $(\omega L_i)/(\omega L_i)_{hf}$ and $L_i/(L_i)_0$ are available[i].

h. L. F. Woodruff Prin of Electric Power Transmission (2nd Ed) (1947) p17

i. Ref a pp 213-4.

k. Ware and Reed Communication Circuits (2nd Ed, 1944) p 8, p 16, etc.

Conductance G

$$G \text{ (mhos/meter)} = \frac{\pi \omega \epsilon_0 \epsilon''}{\cosh^{-1}\left(\frac{s}{d}\right)} \quad (17)$$

$$\epsilon'' = \frac{\sigma_2}{\omega \epsilon_0} \quad (18)$$

$$\tan \Delta = \frac{\epsilon''}{\epsilon'} \qquad \text{loss tangent}^{(i)} \quad (19)$$

For Polyethylene(i) A-3305 insulation data
$\tan \Delta <.0005 \quad 100 \leq f \leq 10^{10}$, $\tan \Delta = .00067$ @ $f = 2.5$ MHz
For paper(i)
$\tan \Delta < .038 \quad f < 10^6$
$\tan \Delta < .066 \quad 10^6 \leq f \leq 10^{10}$

Capacitance C:

$$C \text{ (farads/meter)} = \frac{\pi \epsilon' \epsilon_0}{\cosh^{-1}\left(\frac{s}{d}\right)} \quad (20)$$

Note eq (20) is for solid dielectric correction required for two cylindrical tubes in contact in air.

$$\frac{G}{\omega C} = \frac{\epsilon''}{\epsilon'} = \tan \Delta \quad (21)$$

Eq (21) determines whether G can be neglected.

J. von Hippel QC 585 V6

Sample Calculation of Primary Constants:

Proximity Effect eq (11-1): (compare with p11-58)

$$P = \frac{s/d_1}{\sqrt{(s/d_1)^2 - 1}} = \frac{1.77}{\sqrt{1.77^2 - 1}} = \frac{1.77}{\sqrt{2.13}} = \boxed{1.211} \text{ by (11-1)}$$

#19 gauge copper wire:

p 11-57 $r_0 = \frac{d_1}{2} = 4.45 \times 10^{-4}$ meter

(table for #13, 16, 19, 22, 24 on p 11-56)

$$\frac{s}{d_1} = \frac{.062''}{.035''} = 1.77$$

Terman, ref of 11-58 $\boxed{1.206}$

Bell System Practices

f	c
20 KC	1.06
50 KC	1.13
100 KC	1.18
150 KC	1.204

I don't know whether the variation with frequency is a part of the proximity effect or whether it is due to neglect of the $\sqrt{\text{"Ber Bei"}}$ term of eq (11-2). This can be checked for further accuracy. Article by Dwight giving complete analysis is on order.

For copper: $R_s = 2.61 \times 10^{-7} \sqrt{f}$ (ohms) (MKS

$$R = \frac{2R_s}{\pi d_1} P = \frac{2 \times 2.61 \times 10^{-7} \sqrt{f}}{\pi \; 8.90 \; 10^{-4}} \; 1.211 = 2.28 \times 10^{-4} \sqrt{f} \text{ (ohms/m)}$$

R'(ohms/loop mile) = 1610 R(ohms/meter)

$R' = 1610 \times 2.28 \times 10^{-4} \sqrt{f} = 0.367 \sqrt{f}$ (ohms/loop mile) *

Note transition point between d-c and h f

($.361 \sqrt{f}$ on p 11-57)

D-C Value of R:

$$R_0 = 2 \times 1610 \times 2.773 \times 10^{-2} = 89.2 \text{ ohms/loop mile}$$

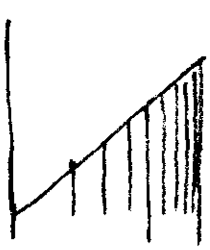
↑ two cond

(ITETG p 821 = 83.8 o

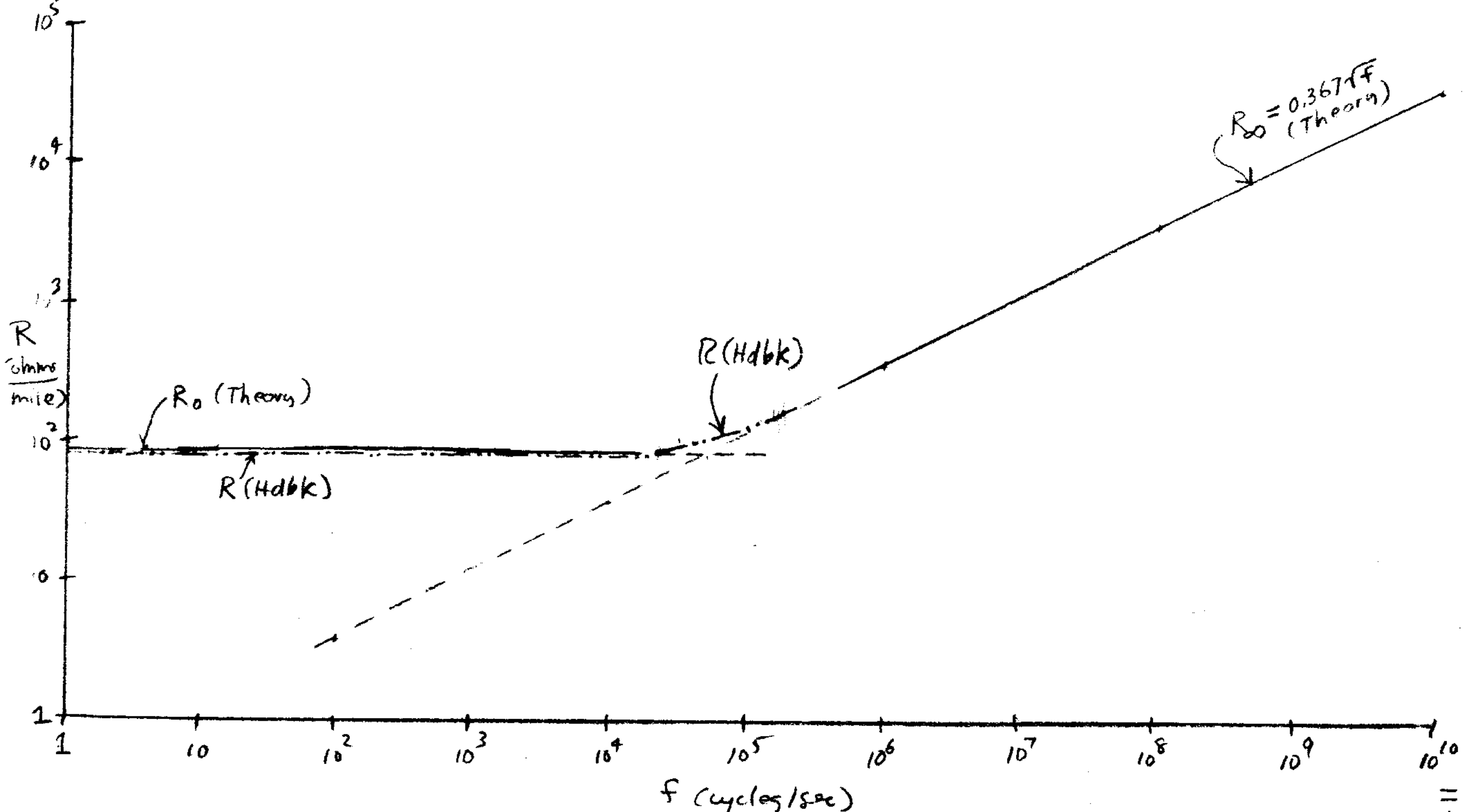
eq (11-9) calc. in part p 11-57

Data from Sequoia Wire, Redwood City

Gauge	Rdc (ohms/loop mile)
#19	92
#22	184
# 24	290



Comparison of Theoretical R for #19 gauge, copper cable (toll) and IT&T Hdbk Values.



Inductance

Eq (11-10) $L_0 = \frac{\mu}{\pi} \cosh^{-1}\left(\frac{s}{d_2}\right) = \frac{4\pi \times 10^{-7}}{\pi} \cosh^{-1}(1.77)$

$= 4\times10^{-7} \cdot 1.17 = 0.469 \times 10^{-6}$ (henries/meter)

$L_0' = 1610 \times 0.469\times10^{-6} = 0.755\times10^{-3}$ (henry/mile)

$L_i = \frac{\mu}{4\pi} = 4\pi\times10^{-7}$ (henry/meter)

(11-11) $L_i' = 1610 \times 4\pi\times10^{-7} = \frac{2.02\times10^{-3}}{1}$ (henry/mile) *

In some reference I have see $L_i = \frac{\mu}{8}$

This would give $L_i'' = 0.25\times10^{-3}$

then $L_0' + L_i'' = 1.005\times10^{-3}$ (henry/mile)

Hdbk value is: $L = 1.112\times10^{-3}$ (henry/mile)

Saquoia value is: $L = 1.003\times10^{-3}$ @ f = 1 kc.

Note that $L_i = \mu$ in ref k p 8 in cgs units.

Woodruff p 17

$$\frac{L}{2} = \left(0.7411 \log_{10}\frac{D}{r} + 0.08047\frac{\mu}{\mu_0}\right)10^{-3} \text{ henry/m}$$

$$L = \left(1.4822 \log_{10}\frac{D}{r} + 0.1609\frac{\mu}{\mu_0}\right)10^{-3} \text{ henry/mile loop}$$

This → $\frac{1}{8}$

These inductances are derived more directly in Smythe p 317, 320

(11-22) $L_{ii}' = \frac{\mu'}{8\pi}$ (internal inductance) (one wire)

(11-23) $L_{ii} = \frac{\mu}{4\pi}\left(1 + 4\ln\frac{b}{a}\right) \rightarrow \boxed{\frac{\mu}{4\pi}\left[1 + 4\ln\left(\frac{2s}{d}\right)\right]}$ (two wires)

$L_i = 1609.4 \times 10^{-7} = \boxed{0.161\times10^{-3} \text{ henry/loop mile}}$

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* See next page

Consider eq (11-14):

$$L_0 + L_i = 4\mu \ln\left(\frac{2a - r_1}{r_1}\right) + \mu$$

$$L_0 = 4 \times 4\pi \times 10^{-7} \ln\left(\frac{S - r_1}{r_1}\right) = 16 \times \pi \times 10^{-7} \ln\left(\frac{.062 - .0175}{.0175}\right)$$

$$= 1.6\pi \times 10^{-6} \ln 2.57 = 1.6\pi \times 10^{-6}\, .97 = 4.87 \times 10^{-6} \text{ h/m}$$

$$L_0' = 1609.4 \times 4.87 \times 10^{-6} = \frac{7.83 \times 10^{-3}}{4\pi} \text{ henry/loop mile} \quad *$$

off by factor of to .4π .625

Consider:

$$L_0 = \frac{\mu}{\pi} \ln\left(\frac{2S}{d}\right) = 4 \times 10^{-7} \ln 3.54 = 0.505 \times 10^{-6} \text{ hen/m}$$

($\ln 3.54$ = 1.262)

$$L_0' = 1609.4 \times 0.505 \times 10^{-6} = .814 \times 10^{-3} \text{ henry/mile}$$

$$L_i' = \frac{\mu}{4\pi} 1609.4 = .161 \times 10^{-3} \text{ henry/mile}$$

$$L_{total} \quad .975 \times 10^{-3} \text{ henry/mile}$$

Consider eq (11-12)

$$L = \frac{4\mu}{\pi} \ln\left(\frac{a + \sqrt{a^2 - r_1^2}}{r_1}\right) = 16\pi 10^{-7} \ln\left(\frac{.031 + \sqrt{.031^2 - .0175^2}}{.0175}\right)$$

$$= 16\pi 10^{-7} \ln 3.23 = 1.6\pi 10^{-6}\ 1.172 = 5.9 \times 10^{-6}$$

$$L' = 1609.4 \times 5.9 \times 10^{-6} = \frac{9.5}{4\pi} \times 10^{-3} \quad \text{wrong units} \quad .755 \quad .749 \times 10^{-3} \quad *$$

Use eq (11-23) for d-c inductance

p11-76

	E_6	L_0	L_i	L_{total}	
Smyth	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{S}{d}\right) + \frac{\mu}{4\pi}$ (high frequency)	0.755	0.161	0.916	
Smyth	$\frac{\mu}{\pi}\ln\left(\frac{2S}{d}\right) + \frac{\mu}{4\pi}$ (low frequency)	0.814	0.161	0.975	$\times 10^{-3}$
Williams	$\frac{\mu}{\pi}\ln\left(\frac{a + \sqrt{a^2 - r_1^2}}{r_1}\right)$ (total for f = 60)			0.749	
	$\frac{\mu}{\pi}\cosh^{-1}\left(\frac{S}{d}\right) + \frac{\mu}{4\pi}\frac{S/d}{\sqrt{(S/d)^2 - 1}}$	0.755	0.195	0.950	

$.031^2 = .00096$

$.0175^2 = .000306$

$.00065$

$r = .0256$

* The equations with incorrect units are off by 4π due to use of unrationalized units in some references.

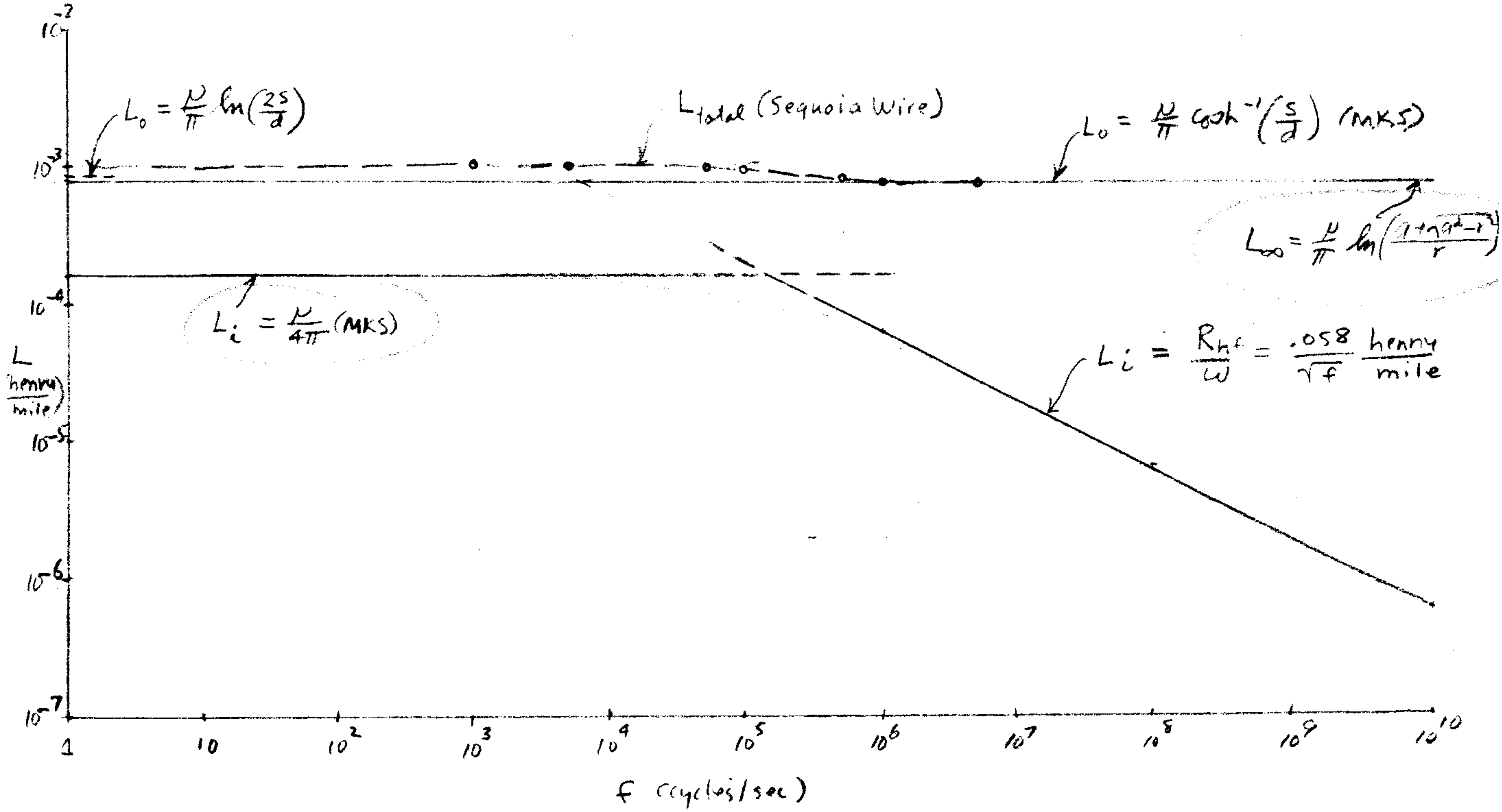
High Frequency Internal Inductance:

$$L_i = \frac{R}{\omega} = \frac{0.367\sqrt{f}}{2\pi \quad f} = \frac{10^{-3}\ 58.4}{\sqrt{f}} \quad \text{henry/loop m}$$

This includes proximity effect for two wires.

Handbook values of L: #19 gauge

ITS T6 Paper Insulation	f (kc)	Sequoia Wire Polyethylene
1,112 mh	0	
	0.1	
1.112	0,5	
1.111	1,0	1,003 mh/m
1,111	1,5	
1,110	2,0	
1,110	3,0	
1,109	5,0	0,990
1.105	10	
1,095	20	
1,055	30	
1,065	50	0.928
1,017	100	0,892
0.980	150	
	200	
	500	0.799
	1000	0,775
	5 000	0,740



L (henry/mile)
f (cycles/sec)
L_o = μ/π ln(2s/d)
L_total (Sequoia Wire)
L_o = μ/π cosh⁻¹(s/d) (MKS)
L_∞ = μ/π ln((a+√(a²−r²))/r)
L_i = μ/4π (MKS)
L_i = R_hf/ω = .058/√f henry/mile

Capacitance and Conductance (C & G)

Eq. (11-20):
$$C = \frac{\pi \epsilon' \epsilon_0}{\cosh^{-1}(\frac{s}{d})} = \frac{\pi\, 2.25 \left(\frac{1}{36\pi} \times 10^{-9}\right)}{1.17} =$$

$$C = \frac{225}{1.17 \times 36} \times 10^{-11} = 5.35 \times 10^{-11} \text{ (farads/meter)}$$

$$C = 1609.4 \times 5.35 \times 10^{-11} = 8.61 \times 10^{-8} = \boxed{.0861 \text{ (}\mu\text{farads/mile)}}$$

polyethylene*

Sequoia Cable

Gauge	C (μfarads/mile)
#19	.083 to .090 ←
#22	.077 to .085
#24	.072 to .080

To calculate for other sizes use Sequoia polyethylene insulation thickness:

19 AWG	.0135"
22 AWG	.0120"
24 AWG	.0100"

(11-17)
(11-19) (11-20)
$$\boxed{G} = \frac{\pi \omega \epsilon_0 \epsilon''}{\cosh^{-1}(\frac{s}{d})} = \boxed{\omega \tan\Delta\, C} = \frac{\pi \omega \epsilon'' \epsilon_0}{\cosh^{-1}(\frac{s}{d})} \quad (11-2$$

$\tan\Delta = .0005$ @ $f = 100$

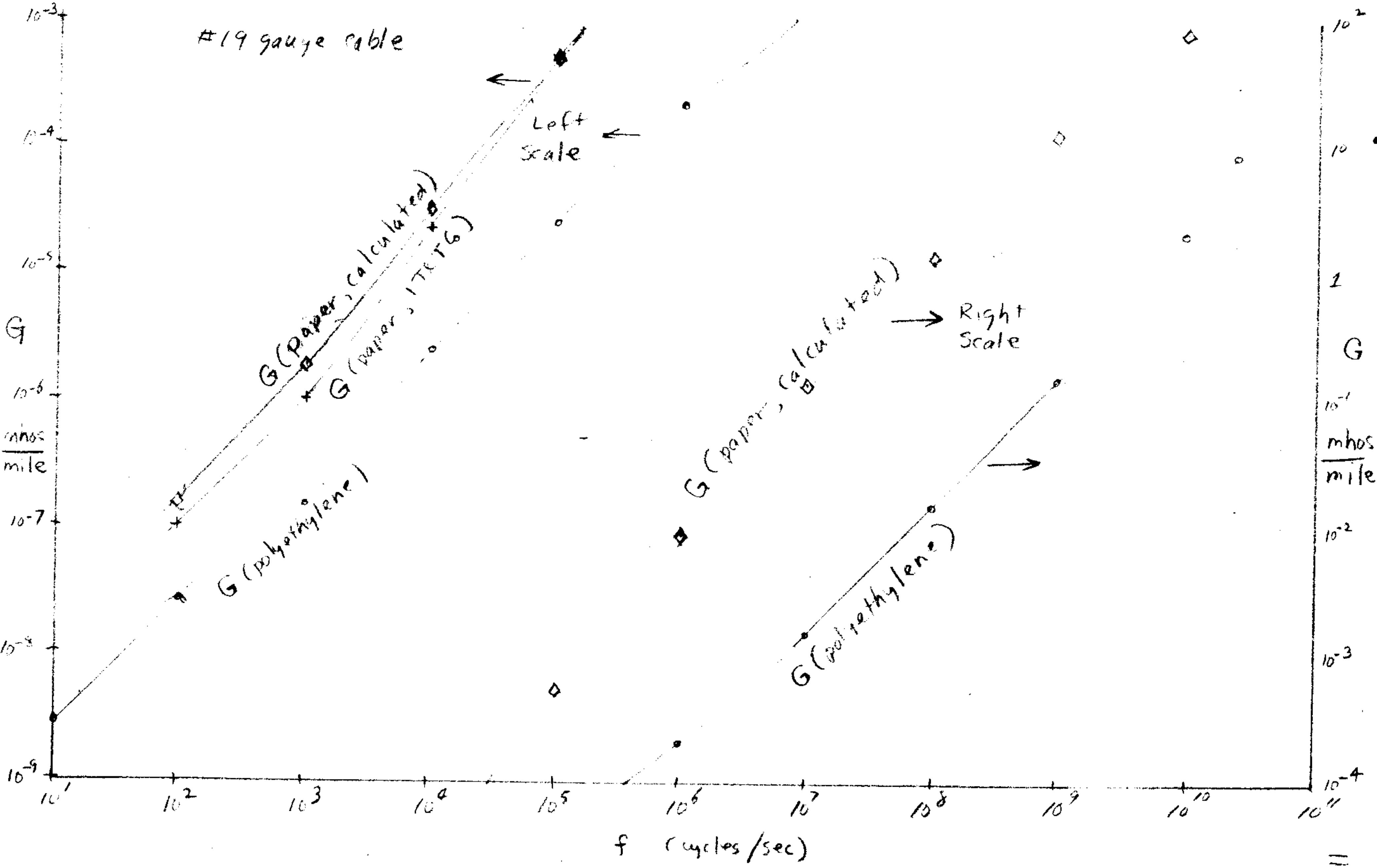
$f = 100$ (cycles/sec) $G = 2\pi\, 100\ .0005 \times .0861 = \mathbf{.027}$ (μmhos/mile) (polyethylene)

For paper $\epsilon' = 3.30$ $\tan\Delta = .0058$

$G_{paper} = .027 \times \frac{.0058}{.0005} = 0.31$ (μmhos/mile) (paper)

(1.75 more wall thickness) ITT paper 0.10

* QC585 V6 Von Hippel A-3305 Polythene 100% polyethylene (DuPont) now replaced by Alathon p 327


#19 gauge cable
Left Scale
Right Scale
G (paper, calculated)
G (paper, ITT TG)
G (polyethylene)
G (paper, calculated)
G (polyethylene)
G
mhos/mile
f (cycles/sec)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{2}
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}

Calculation of G (paper)

$$G = \frac{\pi \omega \epsilon_0 \tan\Delta \, \epsilon'}{\cosh^{-1}(s/d)} = \frac{2\pi^2 \epsilon_0}{\cosh^{-1}(s/d)} [\tan\Delta \, \epsilon' f]$$

$$= \frac{2\pi^2 (\frac{1}{36\pi} \times 10^{-9})}{1.17} [\;] = \frac{100}{18 \times 1.17} \times 10^{-11} [\;] = 4.75 \times 10^{-11} \tan\Delta \, \epsilon'$$

$$G = 1609.9 \times 4.75 \times 10^{-11} [\;] = 7.65 \times 10^{-8} f \epsilon' \tan\Delta \text{ (mhos/mile)}$$

f	ϵ'	$\tan\Delta$	G
10^2	3.30	.0058	0.145×10^{-6}
10^3	3.29	.0077	1.99×10^{-6}
10^4	3.22	.0117	28.9×10^{-6}
10^5	3.10	.020	475×10^{-6}
10^6	2.99	.038	8.71×10^{-3}
10^7	2.86	.057	125×10^{-3}
10^8	2.77	.066	1.4
10^9	2.75	.066	13.9
10^{10}	2.62	.040	80

Exp. values from von Hippel.

What is G at f = 0? This is needed for α_0.

half

C_1 C_2 $C_1 = 2C$ $C_2 = 2C/3$

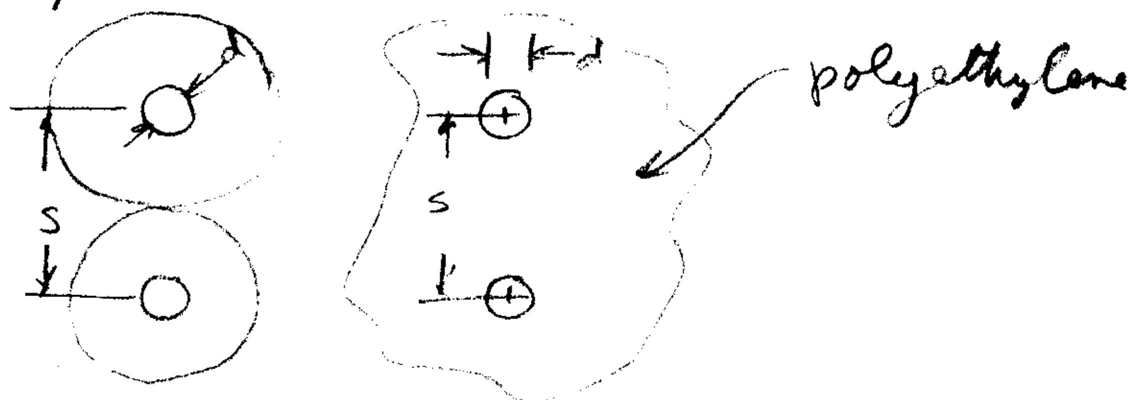
$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{2C \times 2C/3}{2C + 2C/3}$$

$$G = \frac{3}{4} G \qquad C' = \frac{3C}{4}$$

Estimate of G_{d-c}

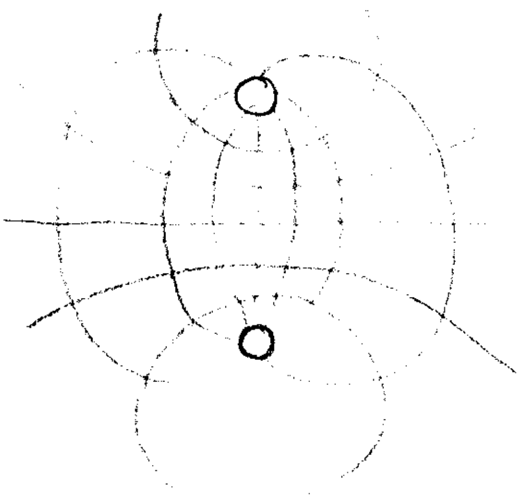
ITETG Hdbk p 67 Polyethylene DE-3401

$\rho = 10^{17}$ ohm-cm @ 25°C



Actual

Assume (because contact area uncertain)



$$C = \frac{q}{V} \qquad R = \frac{V}{I}$$

$$\frac{1}{R} = \frac{I}{V}$$

$$C \longrightarrow \frac{1}{R}$$
$$q \longrightarrow I$$
$$V \longrightarrow V$$

Smythe p234 eq (7) $R = \dfrac{\tau \epsilon_v}{C}$

(8) $R = \dfrac{\tau}{2\pi} \cosh^{-1}\left(\pm \dfrac{s^2 - 2r^2}{2r^2} \right)$ (ohms/cm) (11-25) (ohm-meter)

τ ohm-cm or ohm-meter

If we use $R = \dfrac{\tau \epsilon_v}{C} = \tau \dfrac{\epsilon' \epsilon_0}{\left(\dfrac{\pi \epsilon' \epsilon_0}{\cosh^{-1}(s/d)} \right)} = \dfrac{\tau}{\pi} \cosh^{-1}\left(\dfrac{s}{d} \right)$ (11-26)

(11-25) and (11-26) are equivalent by Smythe § 4.14.

$\rho = 10^{17}$ ohm-cm   Polyethylene

$\tau = 10^{17}$ ohm-cm $\cdot \frac{\text{meter}}{100 \text{ cm}} = 10^{15}$ ohm-meter

$R' = \frac{10^{15}}{\pi} \cosh(1.77) = \frac{1.17}{\pi} 10^{15}$ ohm-meter

$R = \frac{R'}{\ell} \cdot \frac{\text{ohm-meter}}{\text{meter}}$ $R' = 3.72 \times 10^{15}$ ohm-meter

$R = \frac{3.72 \times 10^{15} \text{ ohm-meter}}{1609 \text{ meters/mile}} \geqq 2.32 \times 10^{12}$ ohm-mile

$G \leq \frac{1}{R} \leq 0.432 \times 10^{-12}$ mhos/mile

(at zero frequency)

Paper: Smithsonian Physical Tables (1954)

Table 447

~~$\tau = 2.27 \times 10^{-14}$ mho/cm.~~   d.c

$\tau = 10^{-10}$ 1000 cycles

Note: Tables 420-428 of Smithsonian Phys Tables (1954 Ed) include skin and proximity effect.

Proximity Effect $P = \frac{1 + [G \cdot d^2/s^2]}{F(1 - H d^2/s^2)}$?

F, G, H from tables.

$G_{paper_{d\text{-}c}} = 0.432 \times 10^{-12} \left(\frac{2.27 \times 10^{-14}}{10^{-17}} \right) = 0.980 \times 10^{-9}$ mhos/mile

$G_{paper_{1kc}} = 0.432 \times 10^{-12} \left(\frac{10^{-10}}{10^{-17}} \right) = \boxed{0.432 \times 10^{-5} \text{ mho/mile}}$

$G = G(0) + G(\omega)$

Fig 6 ↑ cw 6B

Polyethylene

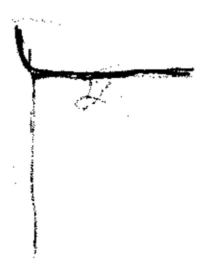
1-79 [illegible] $\triangleq$ [illegible] moles / mole

1-77 [illegible] @ 100 cycles [illegible] inch/in

[illegible] $2.7 \times$ [illegible]

If [illegible]

[illegible]



[illegible]