December 12, 1957
Revised: March 30, 1960

FILE MEMORANDUM: FBW-2.4

SUBJECT: Error Correcting Codes

This old memorandum is reissued for current use in coding studies. The diagram in Fig. 2.4a is useful in discussing "minimum distance."

The list of basic references has been superseded by:

Report RJ-170, "Bibliography on Error Detecting and Error-Correcting Codes," F. B. Wood, June 11, 1959.

F. B. Wood

FBW:pm

2.4 Error Correcting Codes

Basic References:

R. W. Hamming, "Error Detecting and Error Correcting Codes" PAM 1757 (See RJ-170, I, 1950)

C:7-0A Bibliography Section XI

IR-00083 A. P. Fontaine, "Bounds on Probability of Error for Error-Correcting Codes", December 27, 1956 (See RJ-170, II, 1956)
J. E. MacDonald, "A Method for the Construction of Maximum-Minimum Distance Group Alphabets" (Prod. Dev. Pk.) (IBM Journal, Vol. 4, January, 1960, pp. 43-57)
H. S. Shapiro and D. L. Slotnik, "Preliminary Report on Error Correcting Codes", (Computer Lab. Pk.) (See RJ-170, I, 1959)

2.4.1 Note on Character by Character Input

For one character, from Hamming $2^m \leq \frac{2^n}{n+1}$, for single error correcting

Information	Checking	Total
m = 6	k = 4	n = 10 S E C.

For double error detection add one to k

m = 6	k = 5	n = 11 D E D

For one character plus address (64 position):

m = 12 try k = 5 in:

$$2^k \geq m + k + 1$$
$$32 \geq 12 + 5 + 1$$

Observe from PAM 1757 p. 151, Table I, that the number of n's corresponding to each k is:

$$N(n_0, k) = 2^{k-1}$$

For k = 5 n = 11, 12, ..., 26

$$N(11, 5) = 16 = 2^4$$

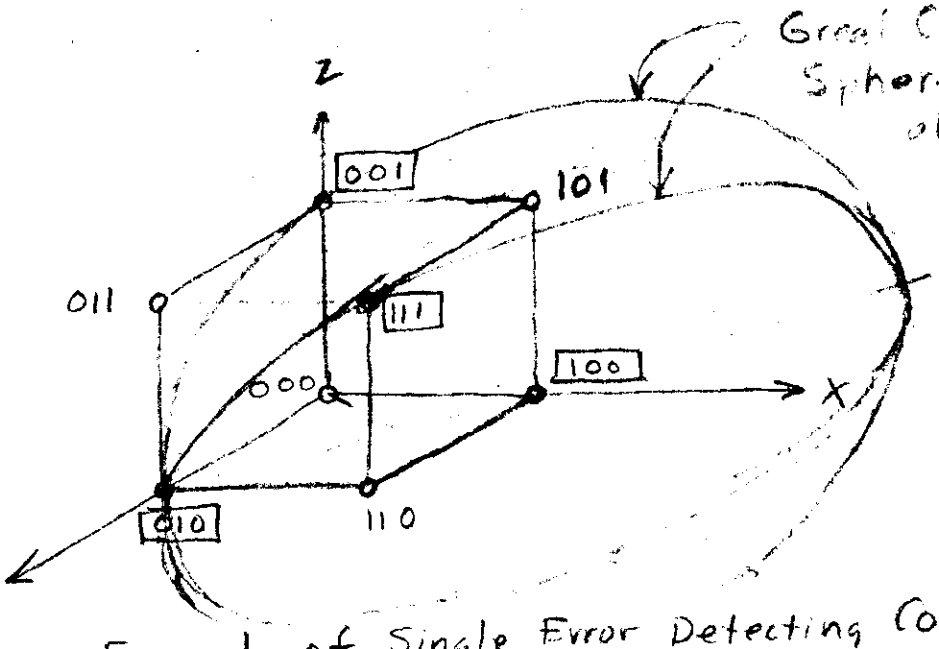

Fig 2,4 a Example of Single Error Detecting Code
Minimum Distance $D(x,y) = 2$

"Radius 2" means distance of two vertices (not radial distance

$r = \sqrt{2} r_0$

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12-12-57

For single error correction, double error detection

m = 12 k = 6 n = 18

If 128 or 256 addresses are required then:

Address (Bits)		Bits/Char	m	k	n
64	(6)	6	12	6	18
128	(7)	6	13	6	19
256	(8)	6	14	6	20

For error-correction separately on character and address the total bits could be M = 22 for 64 addresses.

The problem of the frequency of repeat order in a keyboard input system is discussed in pages 10-23/10-26

2.4.2 Higher Order Error-Correcting Codes

Dr. A. B. Fontaine (Yorktown) has computer programs for generating higher order codes (i.e. large m) and computing number of single, double, triple, etc. error that are detected and/or corrected. **(RJ-170, I, 1958)**

F. B. Wood
12-13-57