5-2

6-20-57

5.1 Decision Rules for Differential Filters

Ref: Marcum and [illegible] "Some Statistical Properties of Signal Plus Narrow Band Noise Integrated over a Finite Time Interval" Jour Appl Phys 27, pp 1442 - 1448, Dec 1956

Proposed Problem. The material of this article would be a good example to translate into decision rules of statistical decision theory.

Application (Ref [illegible]) [illegible] Detection Problems

a) Sampling of 2500 ~ sine wave in 0.4 ms pulsed

$Q = 200 = \frac{\quad}{\Delta f}$ $\Delta f = 12.5$ cycles $= 2$ [illegible]

Noise bandwidth $\beta = \frac{\pi}{2}\Delta f = \frac{39.3}{2}$ cycles

$\beta\delta = $ [illegible] $\times 10^{-3} = \frac{.0157}{2}$ sec [illegible] Q

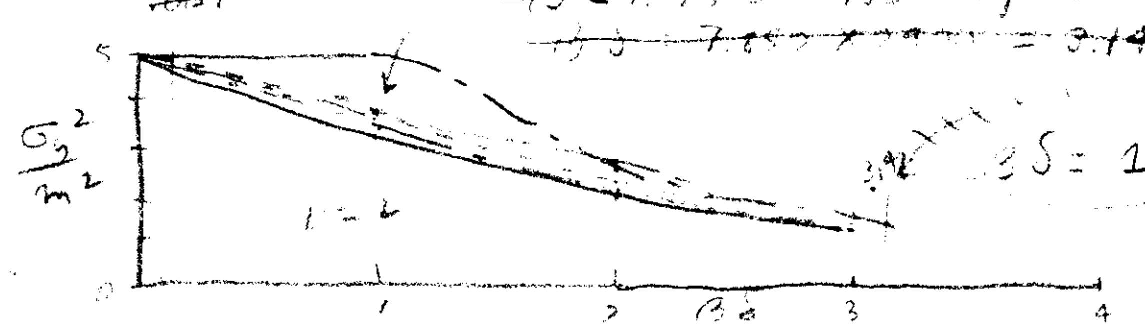
Using line with $\Delta f = 25$ [illegible] Interpolated line [illegible]

~~$\beta = $ [illegible] $= 785$ cycles~~

~~$\beta\delta = $ [illegible] $= 3.14$~~

.0079

~~.0157~~



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From p5-2 we see that for $\beta\delta = \frac{.008}{~~.015~~}$ the High Q circuit doesn't make much difference in $\frac{\sigma_f}{m}$. For no filter, i.e. line itself $\beta\delta = ~~1.4~~$ 1.0. Use big fig 4(d)

$\frac{\sigma_f}{m} = ~~1.6~~$ 1.97* compared with ~~1.5~~ 1.81 for gaussian filter.

(* ideal filter assumed)

For comparing successive bits: $i = 2$

$\beta\delta = .0079$ shift to .01 to use graph 5(d)

$\frac{\sigma_2}{m} = 0.4$ (High Q filter)

$\frac{\sigma_2}{m} = 0.1$ (Gaussian Filter)

$\beta\delta = 1$ (line used as filter ideal)

$\frac{\sigma_2}{m} = 2.5$ (Ideal)

$\frac{\sigma_2}{m} = 1.8$ (High Q*) $\left(Q = \frac{\pi}{2}\frac{f_c}{\beta} = \frac{\pi}{2} \right)$

* Form only

Correspondence between β and Q

$|Y(f)|^2 = 0.5$ or $|Y(f)| = 0.707$ @ $\frac{f^* - f_c}{f_c} = \frac{1}{2Q}$

$|Y(f)|^2 = \frac{1}{1 + [\pi(f - f_c)/\beta]^2}$ gives $f - f_c = \beta/\pi$

$\beta = \frac{\pi}{2}\frac{f_c}{Q} = \pi(f^* - f_c)$