SURVEY OF CABLE CHARACTERISTICS FOR DATA COMMUNICATION

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This survey is a tutorial paper in which material available in scattered sources is brought together and recast in a unified form to simplify the evaluation of alternative cables considered for intra-plant applications by data communication engineers. No attempt is made to obtain the precision needed by cable design engineers, but references are made to the literature available on the accessible more detailed studies.

The channel capacity of some sample data communcation cables are calculated in a series of steps, starting from the physical dimensions of the wire and dielectric. Calculations are illustrated through each step from reviewing the engineering assumptions used in calculating lumped constants, through the influence of the skin effect, and proximity effect in calculating the primary constants of resistance (R), inductance (L), capacitance (C), and conductance (G). The characteristic impedance is then calculated over a large frequency range. Then the secondary constants attenuation (α) and phase (β) are calculated with the assumption that the cables are terminated with their characteristic impedance (Z_0). The equivalent bandwidth (B) is then calculated. Since the channel capacity is dependent upon the bandwidth, signal-noise ratio, and number of states per bit, the influence of crosstalk on the signal-noise ration is discussed. Then the binary noiseless channel capacity is calculated for several cables. Throughout the analysis all parameters are plotted graphically over as large a frequency range as possible to show the different assymptotic formulas which are valid over different frequency ranges.

INTRODUCTION

The computer engineer designing a data communication system to connect remote stations to a central computer faces either of two situations. If he wishes to connect stations that are not in one building or plant site, he may lease communication channels from the telephone companies. But for an isolated system wholly on one customer's plant site, he may have to design the cable system. In the former case, the computer engineer may request the telephone company to provide suitable channels, based upon his bandwidth and noise requirements. In the latter case, the computer engineer must be prepared to select cable of suitable characteristics that can be economically installed.

For intraplant installations, the engineer may use telephone-type twisted pairs at higher frequencies than are normally used, or he may use TV video coaxial cable at a lower frequency than the designed frequency range. To assist him in the design of such installations, the computer engineer presently has access to various tables of characteristics and specifications appearing in the handbooks of cable manufacturers. Theoretical formulas, which are approximations good over specified frequency ranges, are available in various textbooks. But the present treatment of data communcation cables is such that significant facts and formulas are widely scattered in the engineering literature, making it an arduous task for the computer engineer to assemble the information he needs. Moreover, he may be planning to use a cable outside the frequency range for which it was designed and for which manufacturer's data are available. Because the format of the equations for the cable constants is often different for different asymptotic formulas for R and L and for attenuation (α) and phase (β) it is difficient to obtain a simple formula that is correct throughout the transition range. Engineers have experimentally observed as a further complication, that steps appear in the attenuation-versus-frequency curves and in the phase-versus-frequency curves which are not explained in readily accessible handbooks or texts.

The function of this paper is to bring together, from diverse sources, the approximate formulas and sample curves that the computer engineer needs to foresee the problems involved in choosing a data communcation cable. Emphasis is placed on identifying the physical phenomena revelent to the calculation of cable characteristics. As far as possible, the approach is to calculate the primary cable constants-resistance (R), inductance (L), capacitance (C), and conductance (G) from the physical dimensions and physical constants of the materials. The secondary constants attenuation (α) and phase (β) are calculated then from the primary constants and compared with experimental values. This analysis simplifies the determination of the points at which skin effect, proximity, and coupling between pairs become significant factors with twisted pairs in multi-pair cables. In addition, this approach, by presenting a perspective of the full frequency range, shows the frequency range at which the copperweld wire center conductor of coaxial cable, designed for high frequency, causes higher attenuation because the skin depth reaches the thickness the field penetrates the steel core.

These improvements in the treatment of data communication cables are a partial answer to E. G. Fubini's plea for a determined effort toward simplification of concepts, so that the practicing engineer, without bogging down in mathematical theory, may reach higher ground than the blind use of others' formulas.[1] Assisted by the secondary constants estimated in this analysis, a computer engineer can compute the output waveform for a given input waveform and determine whether a proposed cable will transmit the frequency components needed for his application. He can then determine whether the type of signal he is dealing with is limited in bit rate by an equivalent bandwidth, or whether the type of waveform requires a Fourier analysis, followed by a Fourier synthesis using the secondary constants.

Practical data and formulas for primary and secondary constants are available in textbooks such as Creamer's Communication Networks and Lines.[2]

The disadvantage to the computer engineer in using a text like this is that sample curves including the skin and proximity effects are given only for the frequencies currently used for voice and carrier transmission. The objective of this study is to set up a more general perspective so the computer engineer can easily determine the cable characteristics for any potential frequency range being considered.

E. S. Kuh has defined an equivalent bandwidth for networks which are not simple series or shunt LRC circuits, in which the integral of the transfer function (attenuation) over the frequency range is used to obtain an equivalent bandwidth.[3] From the bandwidth one could then calculate the channel capacity by use of the basic formula from C. E. Shannon.[4] G. Raisbeck has calculated the channel capacity for the TEM mode for the high frequency region for Gaussian noise where the skin effect controls the resistance.[5] In data communication problems non-gaussian noise may be encountered, so that a more detailed analysis is needed. Also the desired bit rate may be intermediate to the analyses available for low and high frequency ranges.

In multipair cables the crosstalk can be the limiting factor in determining the channel capacity. The crosstalk between nearby twisted pairs in a cable may become too high before the bit rate reaches the noiseless binary channel capacity. The recent paper by Eager, Jachminowicz, Kolodny, and Robinson on polyethylene insulated telephone cables contains an excellent summary of the crosstalk problem.[6] Their paper also extends the range of curves of primary and secondary constants up to one megacycle per second. Experimental data for the crosstalk from transient pulses in twisted pair cables has been published by Stepheson.[7] Specifications on the capacitive unbalance which contributes to crosstalk are included in Rural Electrification Administration specifications.[8] A recent paper by Partridge includes a curve of attenuation for spiral four cable up to four megacycles per second.[9]

PHYSICAL DIMENSIONS AND CONSTANTS

Sample diagrams of a twisted pair and a coaxial cable are shown in Fig. 1. The computer engineer may use a twisted-pair telephone cable (Fig. 1A) designed originally for voice frequencies and determine its noiseless binary channel capacity, or he may use a coaxial cable designed for the TV video and microwave region and use it at a lower frequency than that for which it was designed.

In the case of the twisted pair, the upper limit may be the bandwidth equivalent to 3db, or the location of a step or change in the secondary constant arising from the skin effect and proximity effect, or it may be limited by the crosstalk in a mulit-pair telephone cable. The usual theoretical formulas for the primary constants of a twisted pair are formulated in a way which neglects the radiation or crosstalk which is negligible at voice frequencies.

The crosstalk at higher frequencies can be avoided by using coaxial cable as shown in Fig. 1E. and 1F. It should be noted that commercially available coaxial cable such as RG-59/U departs somewhat from the ideal shape of Fig. 1G.or 1H.

The cross-section of a cage-type coaxial line is shown in Fig. 1E, where the outer conductor consists of 2N parallel copper wires forming a shielding cage, as shown in Fig. 1G. Manufactured coaxial cable has the outer conductor braided as in Fig. 1F. How close can the engineer predict the behaviour of the cables? Coaxial cable RG-59/U has a copperweld center conductor which consists of a steel wire covered with copper. At the high frequencies for which it is designed, the skin depth is less than the copper thickness, but when such cable is used at lower frequencies attention must be paid to the possible influence of the steel core of the center conductor.

The following four examples listed in Table I are used to illustrate the calculation of cable constants:

1. Polyethylene No. 19 gauge twisted pair
2. Paper strip insultated No. 19 gauge twisted pair
3. Coaxial cable RG-59/U, No. 22 copperweld, 30% conductivity center-conductor, polyethylene, copper braid outer-conductor
4. Idealized solid copper-conductor coaxial cable; approximating cable RG-59/U

The basic formulas in this analysis are given in MKS units, and the sample calculations are given in practical-loop-mile units, abbreviated here as "PLM."

The choice of a cable for intraplant data communication involves a careful consideration of the physical parameters of the cables available. A tinned copper conductor may be convenient at low and intermediate frequencies, but its resistance increases at high frequencies, and the frequency characteristics of standard conductors differ from those of solid conductors.

LUMPED CONSTANT ANALYSIS

Some of the primary cable constants can be derived by a semi-rigorous method which was available before Maxwell's equations were developed. Other constants require rigorous solutions derived from Maxwell's equations with the appropriate boundary conditions. The parts of the primary constants which have transition points where they become a function of a different power of the frequency such as the resistance and internal inductance require solutions of Maxwell's equations in their derivation. The semi-rigorous pre-Maxwellian techniques give accurate enough engineering solutions to the other components: capacitance, external inductance, and conductance. Changes in the frequency dependence of the capacitance and conductance are treated as variations in the complex dielectric constant of the insulating material separating the two conductors. No basic derivations are given in this survey report, but references are made to reliable sources with summaries of the range of validity of the equations. The relationship between the Maxwellian approach involving a complete description of the electromagnetic field and the electrical-engineering (semi-rigorous) approach using lumped constants based on Kirchoff's laws for dc networks has been discussed by R. H. Dicke.[10]

Guillemin has given a semi-rigorous treatment of the longline problem using pre-Maxwellian techniques.[11] The basic derivation for the external inductance is for the dissipationless case; i. e., for zero resistance in the conductor and zero conductance in the dielectric. Under these conditions the proximity effect causes the currents and charges to be concentrated about axes which are eccentric with respect to the conductor axes.

Guillemin evaluates the integrals by substituting finite limits $\pm \Delta Z/Z$, in Figure 2A in place of infinite limits and gives an equation for calculating the maximum error.

The nature of the approximation used in deriving the above is illustrated by Figure 2A. Guillemin obtains a set of difference equations for the change of voltage and current across a length of cable Δz. The difference equations are converted to a set of differential equations by expanding e and i in Taylor series about e_o and i_o and dropping third order terms. The terms in magnetic flux and electric charge are eliminated by integrating the appropriate formulas for magnetic field and electric field from $z = -\infty$ to $z = +\infty$. The current and charge are assumed constant, equal to i_o and q_o respectively. This accuracy of this assumption can be checked by expanding the current in a power series of distance z, and then taking ΔZ to be $\pm \lambda/8$. Only the symmetrical or even power terms contribute to the error. By limiting the assumption of constant current distribution over the interval Δz to a quarter wavelength, the limiting frequency for which this is valid is determined.

Guillemin gives a limiting frequency of 152 megacycles for a pair of No. 10 wires spaced at ten inches. Calculation of this limit for polyethylene No. 19 gauge pair of conductors gives a limiting frequency of 12.6 kilo-megacycles/sec. for five percent error in the lumped constant equivalent circuit of Fig. 1C. This limitation is correct for a single isolated pair of wires. What additional factors must be taken into account when the high frequency range is considered? First the conductance (G) in Fig. 2C must include the dielectric losses which increase with frequency and generally the radiation resistance must be investigated. This radiation resistance at low frequencies can be considered a simple crosstalk problem. The radiation resistance is eliminated in the coaxial cable. A further check on the range of validity of the lumped constant, is to calculate the cut-off frequency for the lowest waveguide mode. For $TE_{n,1}$ coaxial modes the cut-off is:

$$\lambda_c = 2\pi b / n\sqrt{\epsilon'} ,$$

and

$$\lambda_c = 2(b-a)/(m-1)\sqrt{\epsilon'}$$

for $TE_{n,m}$ coaxmode for $m \neq 1$, and for $TM_{n,m}$ - coaxmode. For RG-59/U polyethylene coaxial cable this limit is 108 kilomegacycles per second. A rough estimate of the frequency at which higher modes can propogate in the twisted pair cable is 210 kilomegacycles per second Therefore the lumped constant representation can be used for a single pair and for coaxial cable up to 10 kilomegacycle/sec. with only a few percent error. This limit applies to multi-pair cables if perfectly balanced. In the practical case the crosstalk due to unbalance between pairs requires a correction at a frequency of one megacycle and up.

SKIN EFFECT

Single Solid Copper Conductor

The basic formulas for the skin effect needed for computing the resistance (R) and the internal inductance (L_i) are tabulated in Table II.

The basic format used by Ramo and Whinnery is adopted here, except where new symbols are introduced to provide a formula reasonably accurate for all frequencies.[12]

The definition of skin effect employed follows that given in the American Institute of Physics Handbook: The skin effect is the concentration of high-frequency alternating current near the surface of a conductor.[13]

The skin depth (δ) is defined in Table II, Equation 1 as the depth at which the electric field in a conductor is 0.3679 of its surface value, where f is th frequency in cycles per second, and σ is the volume conductivity in mhos per meter. The surface resistance is the resistance between two edges of a volume bounded by a square of surface area of the conductor having a thickness equal to the skin depth. The permeability of the material is μ. For simplification of the formulas, a normalized radius (q) is used such that it is $\sqrt{2}$ times the radius divided by the skin depth.

The mathematical solutions of the skin effect involve Bessel functions of complex arguments such that the real and imaginary components these functions are defined as ber and bei functions in Equation 4. The skin effect resistance factor is defined by Equation 5 to be equivalent to the factor F used in the Smithsonian Tables.[14] When multiplied by the d-c resistance (R_o), the factor F yields the intermediate frequency resistance. Skin effect transition factors for resistance (T) and for internal inductance (U) are defined by Equations 6 and 7, such that the intermediate frequency surface impedance of a pair of wires is: $Z^i = R_{hf}(T+jU)$. These transition factors are plotted in Fig. 3. All of the above factors except F are plotted in Fig. 3 for Case Number 1 (No. 19 gauge copper wire). Factor F is tabulated in Table 426 of the Smithsonian Tables.

Coated Conductors

The center conductor of Case Number 3, the RG-59/U coaxial cable, is copperweld wire consisting of a steel wire with a 0.002" coating of copper. The use of the copperweld wire at intermediate frequencies complicates the calculation of the resistance and internal inductance because the field penetrates through the copper into the steel. The calculation of the resistance and internal inductance of the copperweld conductor can be approximated by the R and L_i of the flat plate case treated by Ramo and Whinnery.[15] The formulas from this reference have been converted into the format used in this analysis and are tabulated in Table III.

The use of these formulas contains an inherent error through the use of a constant value of permeability (μ_2) of the steel core. At low frequencies as the effect of the copper coating becomes less, the center conductor can be approximated by a solid iron wire. This case has been investigated by M. Kamal Gohar.[16] Gohar's analysis shows the variation of R and L_i with the current level in the wire. For this analysis the permeability is assumed constant.

PROXIMITY EFFECT

The proximity effect is the distortion of alternating current flow in one conductor arising from flow in neighboring conductors. This effect is not present in an ideal coaxial line like Case Number 4 of Table I. The proximity effect can be present in the strands of the braided outer conductor of Case Number 3. However the low resistance of the outer conductor compared to the center conductor makes the proximity effect negligible in this case. If the center conductor were stranded the proximity effect could be significant. A rough estimate of the proximity effect can be estimated from the curves in Terman's Radio Engineer Handbook.[17]

More accurate calculations of the proximity effect can be made by use of Tables 426 and 427 in the Smithsonian Tables.[14]

The proximity factor derived by Dwight is given in Equation 15 in Table IV.[18] The proximity factor is included in the form originally derived by Dwight, in order to show the engineer the nature of the complex series solution. The formulas of Equation 15 as well as the defining Equation 16-20 are reproduced in Table IV in order that the basic formulas may be readily available. The form of the curves and equations in most handbooks are correct, but does not show the nature of the mathematical solution as was done in Dwight's original paper. An alternative representation of the higher frequency proximity effect is given by A. C. Sim.[19] However this is not suitable for this study where the full frequency range is investigated. The proximity effect for the pair of conductors reduces at high frequencies to the form of Equation 21, for which Smythe gives a derivation.[20]

A sample calculation of the proximity effect (P) is plotted in Fig. 3 for No. 19 twisted pair. In this case P rises from 1.0 at 10 kc/s to 1.2 at 1.0 mc/s.

PRIMARY CONSTANTS

Two examples: a twisted pair and a coaxial cable are used to review the primary cable constant. The actual configuration of the two examples are shown in Figs. 1A and 1E. Idealized shapes for use in calculations (approximate) are shown in Figs. 1D and 1H.

Twisted Pair Cable

The primary constants: resistance (R) inductance (L), conductance (C), and capacitance (C) per mile for a twisted pair are summarized in Tables V and VI. Asymptotic formulas have been collected and verified from diverse sources for the different frequency ranges.

The frequency variation of R, C, and L are plotted in Fig. 4 for polyethylene and paper wound twisted pairs. The curves for conductance (G) are plotted in Fig. 5. Curves for $B = \omega C$ are also included in Fig. 5. R and ωL_T have also been plotted in Fig. 5. In this way it can be easily observed which are the dominant terms in different parts of the frequency range for $Z = R + j\omega L$ and $Y = G + j\omega C$. Similarly the same parameters are plotted in Fig. 6 for Case Number 2, Paper Strip Insulated Twisted Pair Cable. An extra curve of G, marked G2 for moist paper is added to show how moisture can make G of significance.

Since the object of this analysis is to provide the data communication engineer with good enough approximations to cable characteristics to evaluate cables for intra-plant use, details needed in the manufacturing design are omitted.

The shielding effect of the other conductors and the reduction of the effective dielectric constant are treated by Maupin.[21] In his study the air in both paper ribbon and paper pulp insulation reduce the effective dielectric constant from 1.75 to approximately 1.50. Similarly, the effective dielectric constant for polyethylene insulated cable is reduced from 2.26 to 1.85. More rigorous mathematical formulas for capacitance than are used in this analysis can be found in Maupin's paper. In this analysis it is assumed that the two dielectric tubes surrounding the two wires are in perfect contact. In reality there is an air gap. Methods of accounting for the air gap are included in a paper by A. S. Windeler.[22]

Idealized Coaxial Cable

Consider the idealized coaxial cable of Fig. 1H with solid copper center and outer conductors. The high frequency formulas can be obtained from one of the handbooks or from a more specialized textbook. Such as Transmission Line Theory by R. W. P. King.[23]

Since coaxial cables are generally used only at high frequencies, intermediate frequency formulas are not included in the usual discussion of coaxial cables. In data transmission and sometimes in TV video transmission coaxial cables are used at frequencies below which the high frequency formulas are exact. The generalization of these formulas to extend through the whole frequency range is summarized in Tables VII and VIII.

The resistance of an idealized coaxial cable is illustrated in Fig. 7 Curves R5 and R6 are the asymptotic values of the outer conductor resistance. Curve R1 and R2 are the inner conductor resistance. The total resistance of the cable is represented by R9 and R11. The inductance is constructed in Fig. 8.

Note that the proximity effect does not enter these formulas due to the symmetry of the perfect coaxial cable. The place where the proximity effect can enter is in the R_i and L_i of the braided outer conductor. In that case it can be shown that the proximity factor is close to unity and at most cannot exceed 1.33 (see Terman, Radio Engineer's Handbook).[24]

The external inductance is given by King.[25] Note that King's treatment does not include the high frequency G, which usually can be neglected.

The resultant primary constants are plotted in terms of $Z = R + j\omega L$ and $Y = G + j\omega C$ in Fig. 9.

Coated Center Conductor and Braided Outer Conductor

The resistance of the copperweld center conductor has been graphically constructed in Fig. 7, by use of curves based on Equation 8 through 11 in Table III as is done by Ramo and Whinnery.[26] The resistance for an iron wire is shown for comparison as curves R3 and R4 in Fig. 7. Curve R1 is the theoretical resistance for copperweld and agrees with the measured d-c resistance of a sample of the RG-59/U cable.

The inductance of the copperweld center conductor is approximated by graphical construction in Fig. 10. Limiting values L1 and L2 for an iron wire, and L5 and L7 for a solid copper wire are given as upper and lower bounds for the inductance. Section L9 in Fig. 10 is an approximation assuming a constant value of magnetic permeability. The sharp corner at the junction of sections L8 and L9 is the intersection of two approximations. Section L8 uses an approximate analysis in which the formulas for a coated flat plate are used for a coated cylindrical wire. Examination of Figs. 7 and 10 shows the detailed calculation of the resistance and inductance of the braided outer conductor will not make a significant difference in the primary constants. The formulas from R. W. P. King are tabulated in Table IX for convenience in calculating the constants for the cage approximation to the braided outer conductor when needed.[27]

The resultant impedance and admittance as functions of frequency for the RG-59/U coaxial cable are plotted in Fig. 11. In this case the conductance G can be neglected.

CHARACTERISTIC IMPEDANCE

To compare the theory with experiment, it is necessary that the experimental lines be terminated with its characteristic impedance. The general formula and asymptotic formulas for d-c, low frequency, and high frequency ranges are listed in Table X.

A sample calculation for No. 19 gauge polyethylene twisted pair is plotted in Fig. 12. In this example the d-c characteristic impedance, Equation 65, applied up to a frequency of 10^{-6} cycles/sec./ or 2.16 cycles/month. The characteristic impedance exhibits an interesting frequency behavior which has been pointed out by Guillemin.[28] At d-c the characteristic impedance has a high value of over four meg-ohms, which decreases as a function of the square root of the frequency to approach an asymptotic value of 110 ohms above 10 kc/s. At about 500 cycles/sec. the impedance is about 600 ohms. During the transition range from 10^{-4} cycles/sec. to 10^{+3} cycles/sec. the characteristic impedance has a phase angle of minus 45 degrees, which looks like a resistance and capacitance in parallel, see Equation 66. At high frequencies above a megacycle/sec., the characteristic impedance is again resistive as is given by Equation 67.

For convenience in appreciating the phase versus frequency curve in Fig. 12, a second scale in time units has been added, showing the period from microseconds, milliseconds, seconds, minutes, hours, etc. The theoretical high value of Z_o at d-c would not be realized until the d-c voltage had been applied for a period of a month. The time constant $T_o = 1/RC$ is 5.3 hours. Note that this corresponds roughly to frequency at which the phase approaches minus 45 degrees.

SECONDARY CONSTANTS
Attenuation and Phase

General Formulas

The basic formulas for attenuation (α) and phase (β) are tabulated in Table XI. Alternative forms are included which become simpler in certain parts of the frequency range. The asymptotic formulas for each appropriate section of the frequency range are listed in Table XII.

Twisted Pair Cables

In Fig. 13, curve $\alpha 1$ is the theoretical curve for No. 19 gauge polyethylene.

Curve $\alpha 2$ is for paper strip insulated twisted pairs of the same gauge. These theoretical curves for two cable pairs could be plotted as one curve with constant to obtain the α and β for different dielectric constants as has been suggested by Fubini.[1]

In Section $\alpha 3$ of the curve, the points (0) are average values for the paper insulated cable published by American Telephone and Telegraph Co.[29] For experimental data on the attenuation of polyethylene insulated cable, refer to the recent paper from General Cable Corporation.[6]

In the higher frequency range of $\alpha 2$, the experimental points $\alpha 4$ depart more from the theory as the frequency increases. In the experimental points the line was terminated in 110 ohms. This departure has not been analyzed The possible causes of the departure are:

(1) Moisture in the cable can increase the value of G by an order of magnitude.

(2) Crosstalk in cable of length over a quarter wavelength long can influence the usefulness of the cable in two ways.

   a. The loss of energy due to radiation to nearby pairs can introduce an additional term in the primary constants, affecting R and G.

   b. The reception of radiated energy from nearby cable pairs reduces the signal-to-noise ratio.

In Fig. 14 curve $\beta 1$ is for polyethylene and curve $\beta 2$ is for paper insulation. There is a difference in the agreement between theory and experiment for α and β .

Examination of Equation 69 and 70 in Table XI indicates the direct into study the discrepancy between theory and experiment at high frequencies in the attenuation for paper insulated cable. A look at Fig. 6 will show that when the paper insulation is moist, the conductance (G) cannot be neglected at high frequencies because G exceeds five percent of the susceptance at 10 kc/s and exceeds 55% of the susceptance at one megacycle/sec.

Coaxial Cable

Curves of attenuation and phase for coaxial cable RG-59/U with copperweld center conductor are plotted in Fig. 15. The attenuation curve 1 and the phase are shown as solid lines where the physical constants used are reliable. They are plotted as dotted lines in the lower frequency range where the approximation was made in the permeability of the steel in the copperweld center conductor. The points ⊗ are typical values of attenuation supplied by the manufacturer.

Twisted Pairs of Different Gauges

The attenuation for polyethylene twisted pairs of three different gauges are plotted in Fig. 16. The dimensions are tabulated in Table XIII. The constants for the approximate curves used to simplify the integrals for the equivalent bandwidth are listed in Table XIV.

BANDWIDTH

Curves of voltage at the receiving end of a cable for one volt input are plotted for different cable lengths in Fig. 17 (A) and (B) these curves do not come close to the universal resonance curve for which the bandwidth is normally defined at the half power point, ~~shown in Fig. 17(C).~~ The classical definition of bandwidth assumes the universal resonance curve, shown in Fig. 18(A) curve "AS." It can be seen that the transfer function of cables do not follow the classical form of "AS." except in the special case of short lines on short circuit or open circuit test in which case Equations 82A and 82B of Table XV would apply as is summarized by Arguimbau.[30] Since the amplitude vs. frequency curves of Fig. 17(A), (B) do not resemble the classical curve, another definition must be found.

The definition of equivalent bandwidth given by E. S. Kuh appear to be the most plausible one to use.[3] The basic formula is given as Equation 83 in Table XV. The straight line segment approximation to the attenuation curves in Fig. 16 are given by Equation 85 to 87. Substitution of these in Equation 84 gives the partial contribution to the bandwidth in Equation 88-90.

In this analysis the A_o coefficient was taken for f = one cycle/sec. instead of zero cycles/sec. The equivalent bandwidths for a ten mile line and a three mile line are shown in Fig. 18(B) and Fig. 18(C). For the ten mile line two different bandwidths are shown in Fig. 18 (B). B is the bandwidth from the definition used in this analysis, based upon integrating from zero to infinite frequency and using the coefficient A_o as the constant at one cycle/sec. For long cables it is more realistic to consider how much of the spectrum is really going to be used. For example, if a high pass filter will cut out the spectrum from zero to 300 cycle/sec, then we have a different bandwidth, B'(300) as is shown in Fig. 18(D). Here B in Fig. 18 (C) ≈ 1.6 kc/s and B in Fig. 18(D) = 3.4 kc/s which is over a factor of two ratio. Therefore to compare the channel capacity curves derived from the bandwidth

B of this analysis, with the established tolerable lengths of cable for telephone transmission without amplification, the bandwidth will have to be recalculated after cutting out the parts of the spectrum that are not used in practice.

CROSSTALK

Sets of parallel wire cable pairs would couple energy between themselves if steps were not taken in the design to reduce crosstalk to a minimum. A single set of parallel wires will radiate energy like an antenna when the length approaches a quarter wavelength. Twisting a pair of wires will almost cancel out the radiation. Twisting each pair in a cable with a different lay will reduce the possibility of two pairs being twisted in synchronism so that energy could be coupled even though the pairs are twisted. Various factors prevent perfect balancing of the pairs in manufacturing, so there is a residual small capacitive unbalance between cable pairs.

The use of twisted pairs in multi-pair cables is limited by the frequency at which the crosstalk from nearby pairs reduces the signal-to-noise ratio below a tolerable level. If this condition is reached at a frequency lower than that at which the attenuation or the equivalent bandwidth is the limiting factor, it may be necessary to use coaxial cable.

The size of the capacitive unbalance tolerated in the manufacturing of cables can be found in specifications of cable users such as the R. E. A. [8] One could calculate a limit on the crosstalk voltage by adding up the voltages induced in the RC circuits consisting of the capacitive unbalances and the Z_o

of the pairs. However as the frequency increases some inductive coupling between pairs becomes significant. For analyzing the crosstalk the equivalent circuit of Campbell is very useful.[31] The crosstalk in polyethylene twisted pair cables has been thoroughly investigated by Eager, Jachminowicz, Kolodny, and Robinson.[6] Their curves of far end crosstalk have frequency variation that is consistent with a simple capacitive unbalance effect. Their near end crosstalk curves resemble approximately the frequency variation one would obtain from a radiation resistance calculation where the radiation wires are partially shielded. For curves of radiation resistance, see R. W. P. King.[32] For the details of the status of crosstalk see the reference cited. The crosstalk due to pulses in short lengths of twisted pair cable has been measured by Stephenson.[7]

BINARY NOISELESS CHANNEL CAPACITY

Shannon's basic formula for the channel capacity of a communication channel is given by Equation 91 in Table XVII.[4,33] For binary transmission and when the signal-to-noise ratio (S/N) is greater than 3 or 9.5 db, Equation 95 becomes the limiting equation. The transition between these two equations has been analyzed by Jelonek and by Watanabe.[34,35] In order to simplify this study, a binary noiseless channel is assumed. For twisted pairs the crosstalk must then be evaluated to determine if the results must be modified on account of the crosstalk noise.

A sample calculation of the channel capacity is shown in Fig. 19 where the partial capacities due to the three frequency bands separated by f_a and

f_b are plotted separately as C*1, C*2, and C*3. Two asymptotic formula which are lower bounds and upper bounds respectively C*4 and C*5 are plotted. The total channel capacity C*T is shown as a solid line.

This calculation of the channel capacity has an assumption which requires closer examination. Namely the use of the coefficient A_o for one cycle/sec. in Equations 88-90. This assumption computes the equivalent bandwidth in terms of the area having the height of the highest point in the amplitude vs. frequency plane as is illustrated in Fig. 18. If we use a waveform that has no frequency components below some specified point or if there is a high pass filter which cuts out the low frequencies, the response at the lowest remaining frequency or some mean frequency must be used to compute A_o.

For example, in Fig. 18(CD) for a 10 mile length of line, the bandwidth are as follows:

$$0 \leq f < \infty, \ B = 1,600 \text{ c/s.}, \ C^* = 3,200 \text{ bps}$$

$$300 \leq f < \infty, \ B = 3,400 \text{ c/s}, \ C^* = 6,800 \text{ bps}$$

This uncertainty can not be resolved by taking a mean value of amplitude, because the amplitude dies off exponentially toward infinite frequency. Dividing by infinitely would give zero mean.

This means that the channel capacity C* plotted in Figures 19-21 is the channel capacity determined without any information on the waveshape to be used for a bit of information. If information on the waveform to be used or the frequency spectrum needed to synthesize it, a more detailed analysis of the bit rate can be made. The basic way to resolve any questions about the channel capacity for a particular waveshape is to use the Fourier Spectrum of the waveshape and perform the convolution with the impulse function of the cable to see at what bit rate the interpulse interference causes error.

Channel capacity curves for Number 19, 22 and 24 gauge polyethylene insulated twisted pair cables are plotted in Fig. 20. The point D is based on an experimental measurement of the rise time on an installed cable.

The channel capacity curves for RG-59/U coaxial line and for Number 19 gauge paper insulated twisted pair are compared in Fig. 21. The coaxial line has the higher channel capacity for short lengths. For long lengths the low frequency components of the electromagnetic field penetrating the steel core of the copperweld outer conductor drops the channel capacity of the RG-59/U coaxial cable to below that of the twisted pair.

CONCLUSIONS

It is possible to start with the physical dimensions and physical constants of communication cables and carry through the successive stages of calculating primary constant characteristic impedance, secondary constants, bandwidth, and channel capacity. As bit rates of data communication are pushed up, the transition region where the skin effect and proximity effect begin to take effect has some interesting properties. This transition causes the attenuation to level off as the frequency is increased and then turn up again proportional to the square root of the frequency. This plateau in the attenuation curve causes the channel capacity to approach two asymptotic values, one for short lines, the other for long lines.

The original form of Dwight's equations for the proximity effect and skin effect are used here to convey a more accurate picture of the mathematical solution than is given by most handbook curves and tables. The form of transition factors equation developed here make it easier to follow the transition in resistance and inductance from low frequencies up to high frequencies.

The theoretical values of attenuation and phase agree well with experimental values except that the error increases with frequency. The characteristic impedance shows an interesting phenomena of having a high resistive value at low frequency slower than a cycle per day, has a 45 degree phase angle from 10^{-4} to 10^{3} cycles/sec. In this transition region the characteristic impedance decreases with the square root of the frequency at 1 megacycle and up the impedance has become resistive again, and has a constant value in the order of 100 ohms.

It is shown that the customary three db down points for determing the bandwidth of a circuit do not fit the case of these transmission lines. The equivalent bandwidth integral of Dr. E. S. Kuh is found to be useful in calculating the binary noiseless channel capacity of cables.

APPENDIX I. SUPPLEMENTARY REFERENCES.

F. W. Grover has published tables for computing the skin and proximity effects. 36 In the case of stranded wires the geometric mean distance formulas of Woodruff may be used in obtaining the low frequency resistance and internal inductance. 37 For coaxial cables some useful formulas are given in the Turbo-Brand Catalog which include the effects of stranded center conductors and braided outer conductors. 38 If one is considering other dielectrics, the extensive tables of dielectric constant and loss tangent in von Hippel's tables are very useful. 39

Other curves of attenuation versus frequency for twisted pair cables are available in Department of the Army Technical Manual TM 11-486-10. 40 The Rome Cable Manual includes some tables of resistance of stranded conductors as a function of frequency and attenuation curves for paper and polyethylene insulated cables up to 360 kc/s. 41 Attenuation curves for Nos. 22, 24, and 26 gauge cables are plotted in an article on television transmission by C. R. Kraus.42 The attenuation for a 3/16" diameter coaxial cable with partially blown-up polystyrene is shown for the five to 15 mc/s region in an article on closed-circuit television by L. G. Schimpf.43

NOMENCLATURE

a	= one-half of the axial separation of a pair of wires
a	= radius of center conductor of coaxial cable
a'	= radius of steel core of copperweld center conductor
a''	= radius of wire in braided shield of coaxial cable
A	= amplitude of voltage at receiving end of line
A_i, B_i, C_i	= complex coefficients in series solution for proximity factor
b	radius of outer conductor of coaxial conductor
B^*	bandwidth (cycles/sec.)
$B = \omega C$	= capacitive susceptance (mhos/mile)
C	= capacitance
C^*	= channel capacity (bits/sec)
C^*_*	= channel capacity, lower bound
C^*T	= total channel capacity
D	= conversion factor from nepers to decibels
d	= diameter of conductor
e_o, e_1, e_2	= voltage between two conductors at points 0, 1, 2.
f, f_a, f_b	= frequency
$F(q)$	= alternative form of transition factor for skin effect, ratio resistance of isolated conductor divided by its dc resistance
G, G_o, G_f	= conductance per unit length total d-c, hf
$H_o^{(1)}(x)$	= Hankel function of the first kind
i_o, i_1, i_2	= current in conductor at points 0, 1, 2

$\mathcal{I}m(x)$	=	imaginary part of X.
j	=	$\sqrt{-1}$
$J_n(x)$	=	Bessel function of the first kind of order n.
K	=	high frequency attenuation - resistance constant (Table XII)
K	=	surface resistance ratio, coated cond.(Table III)
K_1, K_3	=	approximate attentuation coefficient (nepers- $\sqrt{sec}$/ mile)
K_2	=	approximate attenuation coefficient (nepers/ mile)
k_o	=	shape factor for cage transmission line
ℓ	=	length of cable
LX	=	parameter of cable of x miles length
L	=	inductance per unit length
L_i	=	internal inductance due to flux linkages in conductor
L_o	=	external inductance
L_T	=	total inductance per unit length
ℓn	=	natural logarithm
M	=	coated conductor factor
MKS	=	meter-kilogram-second system of units
N	=	one half of the equivalent number of conductors in a shield
N_i	=	partial sum of complex coefficients in the series solution for proximity factor
$p = j^{-1/2}q$	=	complex normalized radius
P (s, q, d)	=	proximity effect ratio, i.e., ratio of resistance of two wires with separation s divided by resistance of two wires with infinite separation.

P_{hf} (s, d)	= asymptotic value of proximity effect at high frequency
PLM	= practical units per loop mile
q	= $\sqrt{2}$ (radius/skin depth) = normalized radius
Q	= conversion factor from MKS to PLM units
r	= radius of conductor
R	= resistance per unit length (ohms/meter) or (ohms/loop mile)
R_{hf}	= high frequency resistance of conductor
R_{if}	= intermediate frequency resistance of conductor
R_o	= d-c resistance of conductor
R_S	equivalent surface resistance of conductor due to skin effect
$\mathcal{R}e(x)$	= real component of X
s	= axial separation of a pair of conductors
s	= equivalent thickness of outer conductor of a coaxial cable
S	= number of available symbols in period T_o
T	= time constant of circuit (sec or other time units)
T_o	= duration of bit period (sec)
T (q)	= transition factor due to skin effect for resistance (change at low frequencies) for round conductor
tan Δ	= loss tangent of dielectric
U(q)	= transistion factor for internal inductance due to change in skin effect at low frequency for round conductor
W(q)	= tubular conductor impedance transition factor
x, y, z	= three dimensional coordinate axes.
$X = \omega L$	= inductance reactance

$Y = G + jB$ = admittance (mhos/mile)

$Z = R + jX$ = impedance (ohms/mile)

Z_o = characteristic impedance (ohms)

Z_i, Z_a^i, Z_b^i = internal impedance of conductor

Z_s = surface impedance

α = attenuation of cable (nepers/meter) or (nepers/mile) or (decibels/mile)

β = phase shift of cable (radians/meter) or (radians/mile)

δ = skin depth

Δ = phase angle of complex dielectric constant

ϵ_o = dielectric constant in free space = $(1/36\pi) \times 10^{-9} \frac{\text{farad}}{\text{meter}}$

ϵ_i = dielectric constant in medium i

θ = phase angle of characteristic impedance

$K = \epsilon_i / \epsilon_o$ = permitivity

λ = wavelength along cable

μ_o = permeability of free space = $4\pi \times 10^{-7}$ (henin/meter)

μ_i = permeability of medium i

σ = volume conductivity (mhos/meter)

τ = coating decay constant

$\omega = 2\pi f$ = angular frequency (radian/sec)

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Table I. Dimensions and Physical Constants of Cables Used for Sample Calculations.

Case Number and Description.	Dimensions and Constants of Conductors: Dimensions	Dimensions and Constants of Conductors: Constants	Physical Constants of Dielectric
1. Twisted Pair, No. 19 Gauge, Polyethylene	d=0.0359" (0.000 83 m.) s=0.0629" (0.00145 m.)	$\mu_1' = 1.0$ $\epsilon_1' = 1.0$ $\sigma_1 = 5.8 \times 10^7$ mho/m	$\epsilon_2' = 2.26$ $\mu_2' = 1.0$ $\sigma_{20}' = 10^{-15}$ mho/m $\tan \Delta = 0.0005$
2. Twisted Pair, No. 19 Gauge, Paper Strip	Same as in No. 1 above.		$\epsilon_2' = 1.75$ $\sigma_{20}' = 2.27 \times 10^{-12}$
3. Coaxial Cable, RG-59/U, Polyethylene Insulated, Copperweld Center Conductor, Braided Copper Outer Cond.	Center Cond. Copperweld a=0.0127" a'=0.0125 Outer Cond. 112 strands No. 34 Copper wire braid b=0.074" a"=0.0032"	30 per cent conductivity $\mu_0/\mu_1 = 250$ $\sigma_0/\sigma_1 = 10$ Same as in No. 1.	Same as in No. 1.
4. Coaxial Cable, Idealized Form Approximating RG-59/U.	Center Cond. Solid Copper a=0.0127" Outer Cond. Copper Tube b=0.074"	Same as in No. 1.	Same as in No. 1.

Table II. Skin Effect Formulas.

Factor	Formula (MKS)	Equation Number
Skin depth	$\delta = 1 / \sqrt{\pi f \sigma}$ (meters)	(1)
Surface resistance .	$R_s = \sqrt{\pi f \mu / \sigma}$ (ohms/square)	(2)
Normalized radius .	$q = \sqrt{2}\ r / \delta = r \sqrt{\omega \mu \sigma}$	(3)
Ber and Bei functions	$J_n(j\sqrt{j}\ q) = \mathrm{ber}_n\ q + j\ \mathrm{bei}_n\ q$	(4)
Skin Effect Resistance Factor .	$F = \frac{R_{1f}}{R_o} = \frac{q}{2} \left[\frac{\mathrm{ber}\ q\ \mathrm{bei}'\ q - \mathrm{bei}\ q\ \mathrm{ber}'\ q}{(\mathrm{ber}'\ q)^2 + (\mathrm{bei}'\ q)^2} \right]$	(5)
Resistance Transition Factor .	$T = \frac{R_{1f}}{R_{hf}} = \sqrt{2} \left[\frac{\mathrm{ber}\ q\ \mathrm{bei}'q - \mathrm{bei}\ q\ \mathrm{ber}'\ q}{(\mathrm{ber}'\ q)^2 + (\mathrm{bei}'\ q)^2} \right]$	(6)
Inductance Transition Factor .	$U = \frac{\omega L_1}{R_{hf}} = \sqrt{2} \left[\frac{\mathrm{ber}\ q\ \mathrm{ber}'\ q + \mathrm{bei}\ q\ \mathrm{bei}'\ q}{(\mathrm{ber}'\ q)^2 + (\mathrm{bei}'\ q)^2} \right]$	(7)

Table III. Skin Effect Formulas for Coated Conductors.

Factor	Formula (MKS units)	Eq. No.
Surface Impedance . .	$Z_s=(1+j)\ R_s \cdot M$	(8)
Coated Conductor Factor	$M=\left[\dfrac{\sinh \tau_1 d + K \cosh \tau_1 d}{\cosh \tau_1 d + K \sinh \tau_1 d}\right]$	(9)
Coating Decay Constant	$\tau_1=(1+j)/\ \delta_1$	(10)
Surface Resistance Ratio	$K = R_{s2}/R_{s1}= \sqrt{\mu_2\sigma_1/\mu_1\sigma_2}$	(11)
Transition Factor for Tubular Conductor	$W = \dfrac{R_{1f} + j\omega L_1}{R_{hf}}$	(12)
Outer Surface . .	$W_o=j^{-1/2}\sqrt{2}\left[\dfrac{J_o(p_o)H_o^{(1)\prime}(p_1)-J_o{}'(p_1)H_o^{(1)}(p_o)}{J_o'(p_o)H_o^{(1)\prime}(p_1)-J_o'(p_1)H_o^{(1)\prime}(p_o)}\right]$	(13)
Inner Surface . .	W_1, interchange p_1 and p_o in eq. 13.	
Normalized Radius of Inner (i) and Outer (o) Surface of Tubular Conductor	$p = j^{-1/2}\ q$	(14)

Table IV. Proximity Effect for a Pair of Conductors with Currents in Opposite Directions.

Factor and Frequency Range	Formulas	Eq. No.
Proximity Factor, Complete Frequency Range	$P=1+\frac{1}{2}\sum_{n=1}^{\infty}\left\|\frac{N_n}{A_o}\right\|^2 \frac{ber_n q\ bei'_n q - ber'_n q\ bei_n q}{ber_o q\ bei'_o q - ber'_o q\ bei_o q}$	(15)
Series Coefficients	$A_o = j\sqrt{j}\ q/2J_1$	(16A)
	$A_n = -(r/s)^n\ j\sqrt{j}\ q/J_{n-1}$, for $n \geq 1$.	(16B)
	$B_n = \sum_{k=1}^{\infty} A_k\left(\frac{r}{s}\right)^{n+k} \frac{(n+k-1)!}{(n-1)!k!}\ \frac{J_{k+1}}{J_{n-1}}$	(17)
	$C_n = \sum_{k=1}^{\infty} B_k\left(\frac{r}{s}\right)^{n+k} \frac{(n+k-1)!}{(n-1)!k!}\ \frac{J_{k+1}}{J_{n-1}}$	(18)
	$N_1 = A_1 + B_1 + C_1 + \ldots\ldots\ldots\ldots$	(19)
	$N_n = A_n + B_n + C_n + \ldots\ldots\ldots\ldots$	(20)
Proximity Factor, High Frequency . .	$P_{hf} = (s/d)/\sqrt{(s/d)^2 - 1}$	(21)

Table V. Primary Cable Constants: R and L_i for a Twisted Pair Cable.

Constant (PLM=Practical Units per Loop Mile)	Frequency Range	Formulas (Internal Factors in MKS Units)	Eq. No.
R Resistance (ohms/mile)	D-C	$R_o' = Q \cdot 2(1/\pi\sigma r^2)$	(22)
	Int.	$R_{if} = Q \cdot (2/\pi d)\ R_s(q)T(q)P(s,d,q)$	(23)
	Low	$R_{if} = Q \cdot R_o F(q)$	(24)
	High	$R_{hf} = Q \cdot (2/\pi d) R_s(q)/\sqrt{1-(s/d)^2}$	(25)
L_i Internal Inductance (henrys/mile)	Low	$L_i = Q \cdot 2(\mu/8\pi)$	(26)
	Int.	$\omega L_i = Q \cdot \{2R_s(q)/\pi d\}\ U(q)\ P(s,d,q)$	(27)
	High	$L_i = R_{hf}/\omega$	(28)

Conversion Factor $Q = 1609.35$ (meters/mile)

Table VI. Primary Constants: L_o, L_T, C, and G for a Twisted Pair Cable.

Constant (PLM Units)	Freq. Range	Formulas (Internal Factors in MKS Units)	Eq. No.
L_T Total Inductance		$L_T = L_o + L_1$	(30)
L_o External Inductance	Low	$L_o = Q \cdot (\mu/\pi) \ln(s/r)$	(31)
	Int.	Smooth Transistion	
	High	$L_o = Q \cdot (\mu/\pi) \cosh^{-1}(s/d)$	(32)
		$L_o = Q \cdot (\mu/\pi) \ln\{(a+\sqrt{a^2-r^2})/r\}$	(33)
C Capacitance (farads/mile)		$C = Q \cdot K\pi\epsilon_o/\cosh^{-1}(s/d)$	(34)
		$C = Q \cdot KC_o/\ln\{(a+\sqrt{a^2-r^2})/r\}$	(35)
G Conductance (mhos/mile)		$G = G_o + G_f$	(36)
	D-C	$G_o = Q \cdot \pi\sigma_o/\cosh^{-1}(s/d)$	(37)
	A-C	$G_f = \omega C \tan\Delta$	(38)
Conversion Factor		$Q = 1609.35$ (meter/mile)	

[redacted]

Table VII. Primary Cable Constants for Coaxial Cable.
(Idealized Coaxial Cable of Fig. 1H.)

Constant (PLM)	Freq. Range	Formula (Internal Factors in MKS)	Inner or Outer	Eq. No.
R Resistance (ohms/mile)	D-C	$R'_{o1} = Q\cdot(1/\pi\sigma a^2)$	inner	(39)
		$R'_{o2} = Q\cdot(1/4N\pi\sigma(a'')^2)$	outer[1]	(40)
		Note 1: Equivalent to double braid, where s is the equivalent solid tube thickness. 2N wires in single braid.		
		$R'_{o2} = Q\cdot(1/2\pi bs\sigma)$	outer	(41)
	Int.	$R_{1f-i} = Q\cdot(R_s(q_1)/2\pi r_1)\,T(q_1)$	inner $r_1 = a$	(42)
		$R_{1f-o} = Q\cdot(R_s(q_1)/2\pi r_1)\,\mathrm{Re}\{W_1(p_o,p_1)\}$	outer $r_1 = b$	(43)
	High	$R_{hf} = Q\cdot R_s(q_a)/2\pi a$	inner	(44)
		$R_{hf} = Q\cdot R_s(q_b)/2\pi b$	outer	(45)
L_1 Internal Inductance (henries/mile)	Low	$L_1 = Q\cdot(\mu/8\pi)$	inner	(46)
		L_1 is the same for outer	outer	(47)
	Int.	$\omega L_1 = Q\cdot(R_s(q_a)/2\pi a)\,U(q_a)$	inner	(48)
		$\omega L_1 = Q\cdot(R_s(q_b)/2\pi b)\,\mathrm{Im}\{W_1(p_o,p_1)\}$	outer	(49)
	High	$\omega L_1 = R_{hf}$	inner and outer	(50)
Conversion Factor		$Q = 1609.35$ (meters/mile)		

Table VIII. Primary Constants: L_o, G, and C for Coaxial Cable.

Constant (PLM units)	Freq. Range	Formulas (Internal Factors in MKS Units)	Eq. No.
L_o External Inductance	High	$L_o = Q \cdot (\mu/2\pi)\ln(b/a)$	(51)
C Capacitance (farads/mile)		$C = Q \cdot 2\pi K \epsilon_o/\ln(b/a)$	(52)
G Conductance (mhos/mile)		$G = G_o + G_f$	(53)
	D-C	$G_o = Q \cdot 2\pi\sigma/\ln(b/a)$	(54)
	A-C	$G_f = \omega C \tan\Delta$	(55)
Conversion Factor		$Q = 1609.35$ (meters/mile)	

Note: $\tan\Delta$ is the loss tangent of the dielectric

Table IX. Primary Constants for Cage Transmission Line Approximation to Coaxial Line with Braided Outer Conductor.

Constant or Condition	Formula	Eq. No.
Equivalence Condition, High Freq.	$a'' = b / 2N$	(56)
Shape Factor	$k_o = \ln(b/a) + (1/2N)\ln(b/2Na'')$	(57)
External Inductance	$L_o = \frac{\mu Q k_o}{2\pi}$	(58)
Internal Impedance	$Z_i = R_i + j\omega L_i = Z_a^i + \frac{Z_b^i}{2N}$	(59)
Center Conductor	$Z_a^i = Q \frac{1+j}{2\pi a} \sqrt{\mu\omega/2\sigma}$ ($)	(60)
Outer Conductor	$Z_b^i = Q \frac{1+j}{2\pi a''} \sqrt{\mu\omega/2\sigma}$	(61)
Capacitance	$C = Q\, 2\pi\epsilon / k_o$	(62)
Conductance	$G = Q\, 2\pi\sigma(\omega)/k_o$	(63)
Conversion Constant	$Q = 1609.35$ (meters/mile)	

$ Note: At low frequencies for copperweld center conductor, Eq. 60 must be modified by use of equations in Table III for skin effect in coated conductors.

Table X. Characteristic Impedance Formulas

Frequency Range	Formula for $Z_o = \lvert Z_o \rvert \angle\theta^\circ$	Eq. No.
General	$Z_o = \sqrt{(R + j\omega L)/(G + j\omega C)}$	(64)
D-C	$Z_o = \sqrt{R/G}$	(65)
Transition	Use general formula	
Intermediate	$Z_o = \sqrt{R/(j\omega C)} = \sqrt{R/\omega C} \angle -45^\circ$	(66)
Transition	Use general formula	
High	$Z_o = \sqrt{L/C}$	(67)

Table XI-A. Secondary Cable Constants: Basic Form and Two Alternative Complete Forms.

Formulas for Attenuation (α) and Phase (β) in (nepers/mile) and (radians/mile) when PLM units are used for R, L, C, and G.	Eq. No.
$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \left\{\frac{1}{2}\left[(R^2+\omega^2L^2)^{1/2}(G^2+\omega^2C^2)^{1/2} \pm (RG-\omega^2LC)\right]\right\}^{1/2}$$	(68)
$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \sqrt{RG}\left\{\frac{1}{2}\left[\left(1+\frac{\omega^2L^2}{R^2}\right)^{1/2}\left(1+\frac{\omega^2C^2}{G^2}\right)^{1/2} \pm \left(1-\frac{\omega^2LC}{RG}\right)\right]\right\}^{1/2}$$	(69)
$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \omega\sqrt{LC}\left\{\frac{1}{2}\left[\left(1+\frac{R^2}{\omega^2L^2}\right)^{1/2}\left(1+\frac{G^2}{\omega^2C^2}\right)^{1/2} \mp \left(1-\frac{RG}{\omega^2LC}\right)\right]\right\}^{1/2}$$	(70)

Table XII. Asymptotic Formulas for Secondary Constants.

Secondary Constant	Frequency Range	Formula (R, L, C, G in PLM units)	Eq. No.
Attenuation (db/mile)	D-C	$\alpha = D\sqrt{RG}$	(71)
	Low	$\alpha = D\sqrt{\omega R_o C/2}$	(72)
	Interm.	$\alpha = D\sqrt{\omega^2 LC/2}\sqrt{\sqrt{1+(R/\omega L)^2} - 1}$	(73)
	High	$\alpha = D\,(K/2)\sqrt{\omega C/L_o}$	(74)
		where $K=R_{hf}/\sqrt{\omega}$,	
		and $D = 8.686$ (db/neper)	(76)
Phase (radians/mile)	D-C	$\beta = 0$	(77)
	Low	$\beta = \sqrt{\omega R_o C/2}$	(78)
	Interm.	$\beta = \omega\sqrt{LC}\sqrt{(\sqrt{1+(R/\omega L)^2} + 1)/2}$	(79)
	High	$\beta = \omega\sqrt{LC}$	(80)

Table XIII. Dimensions of Polyethylene Insulated Twisted Pair Cables Used in the Attenuation Calculations.

Wire Gauge	Wire Diameter	Axial Separation	Polyethylene Wall Thickness	Attenuation Curve on Fig. 16
No. 19	0.0359"	0.0629"	0.0135"	A19
No. 22	0.02535"	0.04935"	0.0120"	A22
No. 24	0.02010"	0.04010"	0.0100"	A24

Table XIV. Straight Line Segment Approximation to the Attenuation of Polyethylene Insulated Twisted Pair Cables.

Wire Gauge	K_1 $\left(\frac{\text{nepers-}\sqrt{\text{sec}}}{\text{mile}}\right)$	K_2 $\left(\frac{\text{nepers}}{\text{mile}}\right)$	K_3 $\left(\frac{\text{nepers-}\sqrt{\text{sec}}}{\text{mile}}\right)$	f_a (cycles/sec.)	f_b (cycles/sec.)
No. 19	0.00503	0.392	0.00190	5 600	50 000
No. 22	0.00648	0.761	0.00207	14 400	140 000
No. 24	0.00801	1.152	0.00248	23 000	200 000

Table XV. Bandwidth Formulas.

Description	Formulas	Eq. No.
Time Constant of Dielectric	$T = R'C = C/G_o$	(81)
Bandwidth of Short-Circuited Cable, Short Length, Low Frequency	$B = R/2\pi L$	(82A)
Bandwidth of Open-Circuited Cable, Short Length, Low Frequency	$B = G_o/2\pi C$	(82B)
Bandwidth of Matched Line, General Case	$B = (1/4\pi A_o) \int_{-\infty}^{+\infty} A(\omega)\, d\omega$	(83)
	$B = (1/2\pi A_o) \int_{0}^{\infty} A(\omega)\, d\omega$	(84)
	where $A(\omega) = e^{-\alpha l}$	
	$A_o = A(0) \simeq A(1/2\pi) = e^{-K_1 l}$	
Approximations Used for α in $A(\omega)$	$\alpha_1 = K_1\sqrt{f}$, $f_a > f$	(85)
	$\alpha_2 = K_2$, $f_a \leq f < f_b$	(86)
	$\alpha_3 = K_3\sqrt{f}$, $f_b \leq f$	(87)

Table XVI.B. Integration of Terms in Bandwidth Formula.

Description	Formulas	Eq. No.
Bandwidth	$2\pi A_o B = \int_0^{f_a} A(\omega)d\omega + \int_{f_a}^{f_b} A(\omega)d\omega + \int_{f_b}^{\infty} A(\omega)d\omega =$	
	$B = B_1 + B_2 + B_3$	
Partial Contributions to Bandwidth	$B_1 = (2/A_o(K_1 l)^2)\left\{1-(1+K_1 l\sqrt{f_a})e^{-K_3 l\sqrt{f_a}}\right\}$	(88)
	$B_2 = (1/A_o)e^{-K_2 l}(f_b - f_a)$	(89)
	$B_3 = (2/A_o(K_3 l)^2)(1+K_3 l\sqrt{f_b})e^{-K_3 l\sqrt{f_b}}$	(90)

Table XVII. Channel Capacity.

Description	Formula	Eq. No.
General Formula for Channel Capacity	$C^* = B \log_2(1 + S/N)$	(91)
Noiseless Channel Capacity with S Available Symbols of Duration T_o (sec.)	$C^* = (1/T_o) \log_2 S$	(92)
Noiseless Channel Capacity when Bandwidth Limited	$B = 1/2T_o$	(93)
	$C^* = 2B \log_2 S$	(94)
Binary Noiseless Channel Capacity	$S = 2$	
	$C^* = 2B$	(95)
	$C^* = C^*T = C^*1+C^*2+C^*3$	(96)

Table **XVIII.** Comparision of Theoretical Channel Capacity with Experimental Data and Telephone Practice.

Property	Gauge	Theoretical Value	Experimental Value	Value Used in Telephone Practice
Rise Time, $T_o = 1/(C^*T)$, 0.79 mile twisted pair	mixed No. 22 and 24	0.86 microsec.	0.91 microsec.	
Maximum Length of Twisted Pair without Repeater for Telephone Use.		Length for Upper Bound on Channel Capacity C^*T of 2400 c/s.		
	No. 19	23 miles		22 miles
	No. 22	19 miles		17 miles
	No. 24	16 miles		14 miles

LIST OF CAPTIONS

Fig. 1. Sample Physical Dimensions of Cables.

(A.) Cross Section of Twisted Pair.

(B.) Side View Twisted Pair.

(C.) Side View, Parallel Approximation to Twisted Pair.

(D.) Cross Section of Approximation, Assuming the Entire Space Surrounding the Conductors is Solid Dielectric (such as Polyethylene).

(E.) Cross Section of Coaxial Cable with Coated Center Conductor, and Braided Outer Conductor.

(F.) Side View of Braid Outer Conductor.

(G.) Side View of Cage Type Transmission Line as an Approximation to the Braided Outer Conductor.

(H.) Cross Section of Idealized Coaxial Line Solid Copper Inner and Outer Conductors.

Fig. 2. Lumped Constant Representation of the Distributed Constant Transmission Line.

(A.) Coordinate System Used in Establishing Engineering Approximation to Obtain Lumped Constants.

(B.) Ideal Twisted Pair.

(C.) Lumped Constant Representation of a Section of the Line.

(D.) Cascading a Series of Lumped Constant Sections to Represent a Long Line.

Fig. 3. Skin and Proximity Effect Factors for No. 19 Gauge Copper Twisted Pair Cable.
$q = \sqrt{2} \cdot$ (Radius/Skin Depth), corresponds to argument in tables of ber and bei functions.

r = Radius (meters).
s = Separation of centers of conductor (meters).
R_s = Surface resistance, copper (ohms/square).
P = Proximity factor.
T = Transition factor for Resistance.
U = Transition factor for Inductance.

Fig. 4. Resistance, Capacitance and Inductance of a Cable Pair, No. 19 Gauge.
R_o = D-C Resistance, Eq. (22).
R1 = High Frequency Resistance with Infinite Separation, Eq. (23) with T=1, P=1.
L1 = Internal Inductance of Two Wires, Low Frequency, Eq. (29).
L2 = Internal Inductance of Two Wires High Frequency, Infinite Separation, Eq. (27) with U=1, P=1.
L3 = Internal Inductance, Intermediate Frequency, S=0.0629", Eq. (7), (15), (27).
L4 = Internal Inductance, High Frequency Proximity Pair, Eq. (21), (28), U=1.
L5 = External Inductance, Low Frequency Uniform Current Distribution in Conductor Exact Solution, Eq. (22).
L6 = Total Inductance for High Frequency (Approximation), Also valid for Zero frequency dissipationless case, Eq. (24)(25).
L7 = Total Inductance, Low Frequency, Eq. (26)(29)(33).
L8 = Total Inductance, Intermediate Frequency, Eq. (7)(15)(27)(24)(25)(33)
C1 = Capacitance, No. 19 gauge, Twisted Pair, with Polyethylene Insulation, Calculated from Eq. (34) with K from von Hippel.

C2 = Capacitance, No. 19 gauge, Twisted pair with Paper Strip Insulation. Frequency variation of K based upon Royal grey paper in von Hippel.

R_2 = High Frequency Resistance, S = 0.0629", Eq. (25).

R_3 = Intermediate Freq. Resistance, Eq. (23).

Fig. 5. Resistance, Inductance, Conductance, and Capacitive Susceptance of a Cable Pair (#19 ga.).

B = Capacitive Susceptance of Polyethylene Insulated Cable.

G = Conductance

G' ~~H~~ = Conductance Calculated from vonHippel's tables.

R = Resistance.

X = Inductive Reactance

X1 = Inductive Reactance at Low Frequencies.

X2 = Inductive Reactance at High Frequencies.

Fig. 6. Resistance, Inductance, Conductance, and Capacitive Susceptance of a Cable Pair (#19 ga.)

B1 = Capacitive Susceptance of Strip Paper Insulated Cable, Using Frequency Variation of Dielectric Constant for Royal Grey Paper from vonHippel's Tables

B2 = Capacitive Susceptance, Assuming Constant Dielectric Constant.

G1 = Conductance of Strip Paper Insulated Cable, Dry.

G2 = Conductance of Strip Paper Insulated Cable, Moist.

H = Conductance Calculated from vonHippel's Tables for Royal Grey paper.

R = Resistance

X = Inductive Reactance

X1 = Low Frequency Inductive Reactance

X2 = High Frequency Inductive Reactance

Fig. 7. Resistance of Coaxial Cable, RG-59/U
R1 = Resistance of Solid Copper Center Conductor, Low Frequency.
R2 = High Frequency Resistance of Solid Copper, Copperweld Center Conductor.
R3 = Upper Bound on Center Conductor Resistance for Solid Iron Wire.
R4 = High Frequency Resistance of Iron Wire Center Conductor, Assuming Constant Permeability.
R5 = Low Frequency Resistance of Outer Conductor.
R6 = High Frequency Resistance of Outer Conductor.
R7 = Low Frequency Resistance of Copperweld Center Conductor.
R8 = Intermediate Frequency Resistance, Copperweld Center Conductor.
R9 = Total Resistance of Idealized Coaxial Cable with Solid Copper Conductors.
R10 = Total Resistance of Copperweld Center Conductor with Solid Copper Outer Conductor.
R11 = Total Resistance, High Frequency.

Fig. 8. Inductance of Coaxial Cable with Solid Copper Center and Outer Conductors.
L1 = External Inductance, Low Frequency.
L2 = Total Inductance, High Frequency.
L3 = Internal Inductance, Low Frequency, Each Conductor.
L4 = High Frequency Interval Inductance, Outer Conductor.
L5 = High Frequency Interval Inductance, Inner Conductor.
L6 = Interval Inductance, Both Conductors.
L7 = Total Inductance, Low Frequency.
L8 = Total Inductance, Intermediate Frequency.

Fig. 9. Resistance, Inductive Reactance, Conductance, and Capacitive Susceptance of Idealized Coaxial Cable, RG-59/U with Solid Copper Conductors.
B = Capacitive Susceptance of Polyethylene Insulated (Solid) Coaxial Line.
G = Conductance of Polyethylene Insulated Coaxial Line.
R = Resistance of Coaxial Line, Solid Copper Conductor.
X = Inductive Reactance of Coaxial Line, Solid Copper Conductors.
X1 = Inductive Reactance, Low Frequency Asymptotic Value.
X2 Inductive Reactance, High Frequency Asymptotic Value.

Fig. 10. Inductance of Coaxial Cable RG-59/U with Copperweld Center Conductor.
L1 = Low Frequency Inductance of Steel Center Conductor without Copper Coating.
L2 = High Frequency Inductance of Steel Center Conductor, Assuming Constant Permeability and Neglecting Saturation Effects of B-H Curve.
L3 = High Frequency, Total Inductance.
L4 = Low Frequency, External Inductance.
L5 Low Frequency, Internal Inductance of Outer Conductor.
L6 = High Frequency, Internal Inductance of Outer Conductor.
L7 = High Frequency, Internal Inductance of Center Conductor.
L8 = Intermediate Frequency, Internal Inductance of Copperweld Center Conductor.
L9 = Approximation to Low Frequency Internal Inductance of Copperweld Center Conductor.
L10 = Internal Inductance Due to Both Center and Outer Conductor.
L11 = Intermediate Frequency Total Inductance.
L12 = Approximation to Low Frequency Total Inductance.

Fig. 11. Resistance, Inductive Reactance, Conductance and Capacitive Susceptance of Coaxial Cable, RG-59/U, with Copperweld Center Conductor and Braided Outer Conductor.

B = Capacitive Susceptance of Polyethylene Insulated (Solid) Coaxial Line.

G = Conductance of Polyethylene Insulated Coaxial Line.

R = Resistance of Coaxial Line, Copperweld Center Conductor.

X = Inductive Reactance of Coaxial Line, Copperweld Center Conductor.

X1 = Inductive Reactance, Low Frequency Asymptotic Value.

X2 = Inductive Reactance, High Frequency Asymptotic Value.

Fig. 12. Amplitude and Phase of Characteristic Impedance of Polyethylene Insulated No. 19 Gauge Cable Pair.

θ = Phase

Z1 = Direct Current $|Z_0|$, Eq. (65).

Z2 = Low Frequency $|Z_0|$, Eq. (66).

Z3 = High Frequency $|Z_0|$, Eq. (67).

Fig. 13. Attenuation of No. 19 Gauge Twisted Pair vs. Frequency.

A1 = Polyethylene Insulated Cable, Theoretical.

$\alpha 2$-$\alpha 4$ = Paper Insulated Cable.

$\alpha 2$ = Theoretical Curve

$\alpha 3$ (o) = Typical Values from A. T. &. T. Co. "Green Book"

$\alpha 4$ (●) = Experimental Values, Strip Paper Insulation (Humidity not known).

Fig. 14. Phase of No. 19 Twisted Pair vs. Frequency.

$\beta 1$ = Theoretical Curve for Polyethylene Insulated Cable.

$\beta 2$-$\beta 4$ = Paper Insulated Cable.

$\beta 2$ = Theoretical Curve

$\beta 3$ = Typical Value from A. T. &T. Co. "Green Book".

$\beta 4$ = Experimental Values, Strip Paper Insulation.

Fig. 15 Attenuation and phase for Coaxial Cable RG-59/U with Copperweld Center Conductor.

$\alpha 1$ = Attenuation, Theoretical Curve.

$\alpha 2$ = Attenuation, Typical Values from Manufacturer (Amphenol Co.)

β = Phase, Theoretical.

Fig. 16. Attenuation of Polyethylene Insulated Twisted Pair Cables (Theoretical Curves).

$\alpha 19$ = No. 19 Gauge.

$\alpha 22$ = No. 22 Gauge.

$\alpha 24$ = No. 24 Gauge.

K1, K2, K3 are straight line approximations to attenuation curves with junctions at frequencies, f_a and f_b.

Fig. 17. Amplitude of Voltage at Receiving End of No. 19 Gauge Twisted Pair, Polyethylene Cable for Sending End Amplitude of 1.0.

(A) Expanded Scale for Region Below 10 kc/sec.

(B) Linear Scale up to One Megacycle/sec.

L0.1 to L10.0 Indicates Line Length of 0.1 mile to ten miles.

Fig. 18. Equivalent Bandwidth of Cable

(A) Comparison of the frequency response of one and ten mile matched cables with a short length of short circuited cable, polyethylene, No. 19 gauge twisted pair

A1 = Receiver end amplitude response for one mile length cable,
A10 = Same for ten mile cable
B1, B10 = Bandwidths for half power prints.
AS = Input current amplitude vs. frequency for a short length of short circuited cable
BS = Bandwidth for half power point

(B) Frequency response curve for a three-mile matched cable.

(C) Frequency response curve for a ten-mile matched cable.

B = Equivalent bandwidth by area method
B10 = Bandwidth by half power point method.

(D) Frequency response of ten mile cable with high pass filter with cut-off at 300 cyn/sec.
A10S = Receiving end amplitude response
B = Bandwidth by area method
BS = Bandwidth between half power points.

Fig. 19. Noiseless Binary Channel
Capacity of No. 19 Polyethylene
Insulated Twisted Pair Cable.
C*T = Channel Capacity vs. Length
C*i = Contribution to Channel Capacity
from different Parts of the Frequency
Range Shown in Fig. 17
C*1 = Partial Channel Capacity in
Range K1, f f
C*2 = Partial Channel Capacity in
Range K2, $f_a < f < f_b$

C*3 = Partial Channel Capacity in Range K3, $f > f_b$.
C*4 = Low Bound on Channel Capacity.
C*5 = Upper Bound on Channel Capacity.

Fig. 20. Channel Capacity of Different Gauges of Polyethylene Insulated Twisted Pairs.
C*19, C*22, C*24 = Upper Bounds on Channel Capacities for Gauges 19, 22, 24 Respectively.
C^*_*19, C^*_*22, C^*_*24 = Lower Bound on Channel Capacities for Gauges 19, 22, 24 Respectively.

Fig. 21. Channel Capacity of Miscellaneous Cables.
C*59, C^*_*59 = Upper and Lower Bounds for RG-59/U Coaxial Cable, Polyethylene Insulation. Copperweld Center Conductor.
C*192, C^*_*192 = Upper and Lower Bounds for No. 19 Gauge, Paper Ribbon Insulated Twisted Pair.

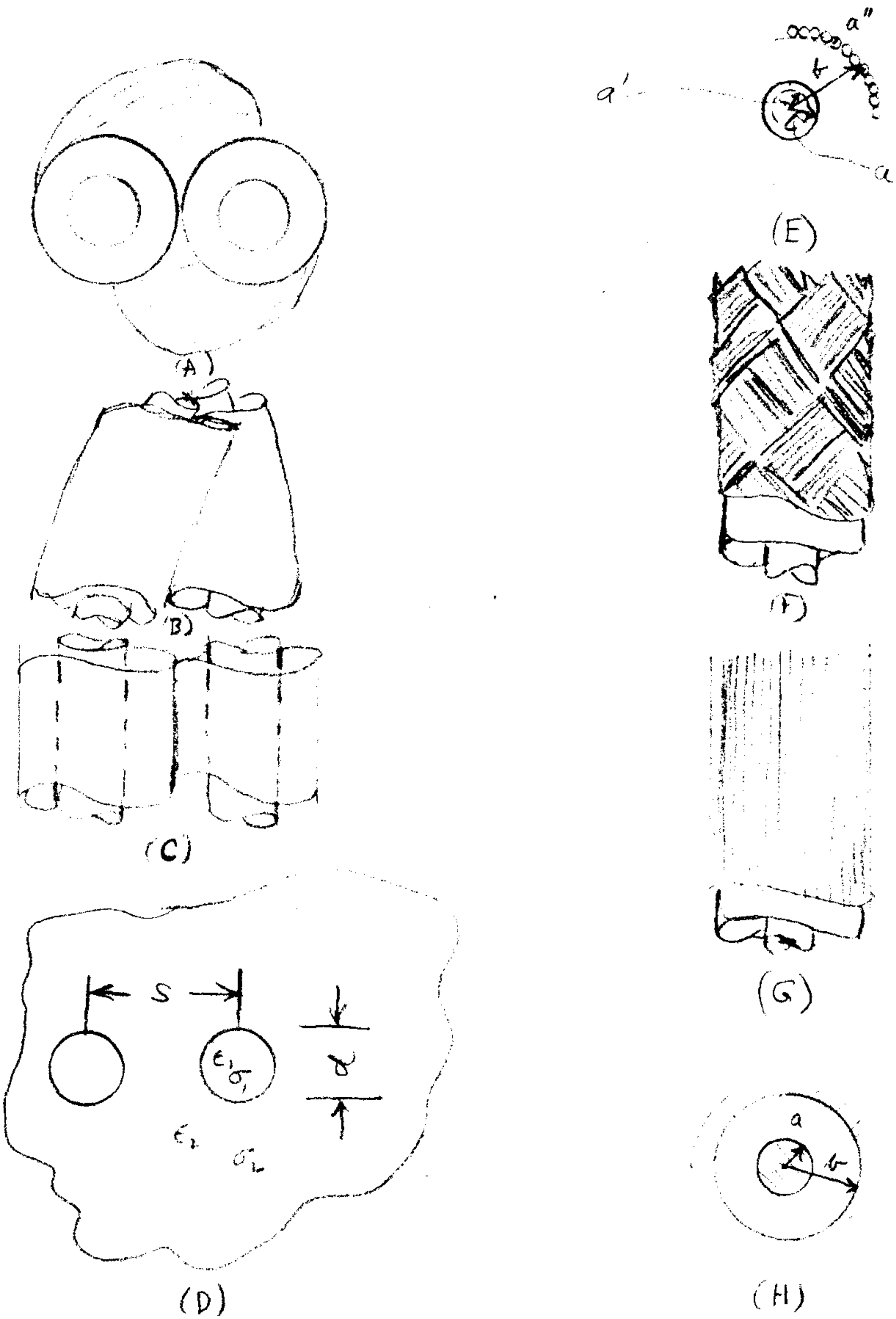

Figure 1. Sample Physical Dimensions

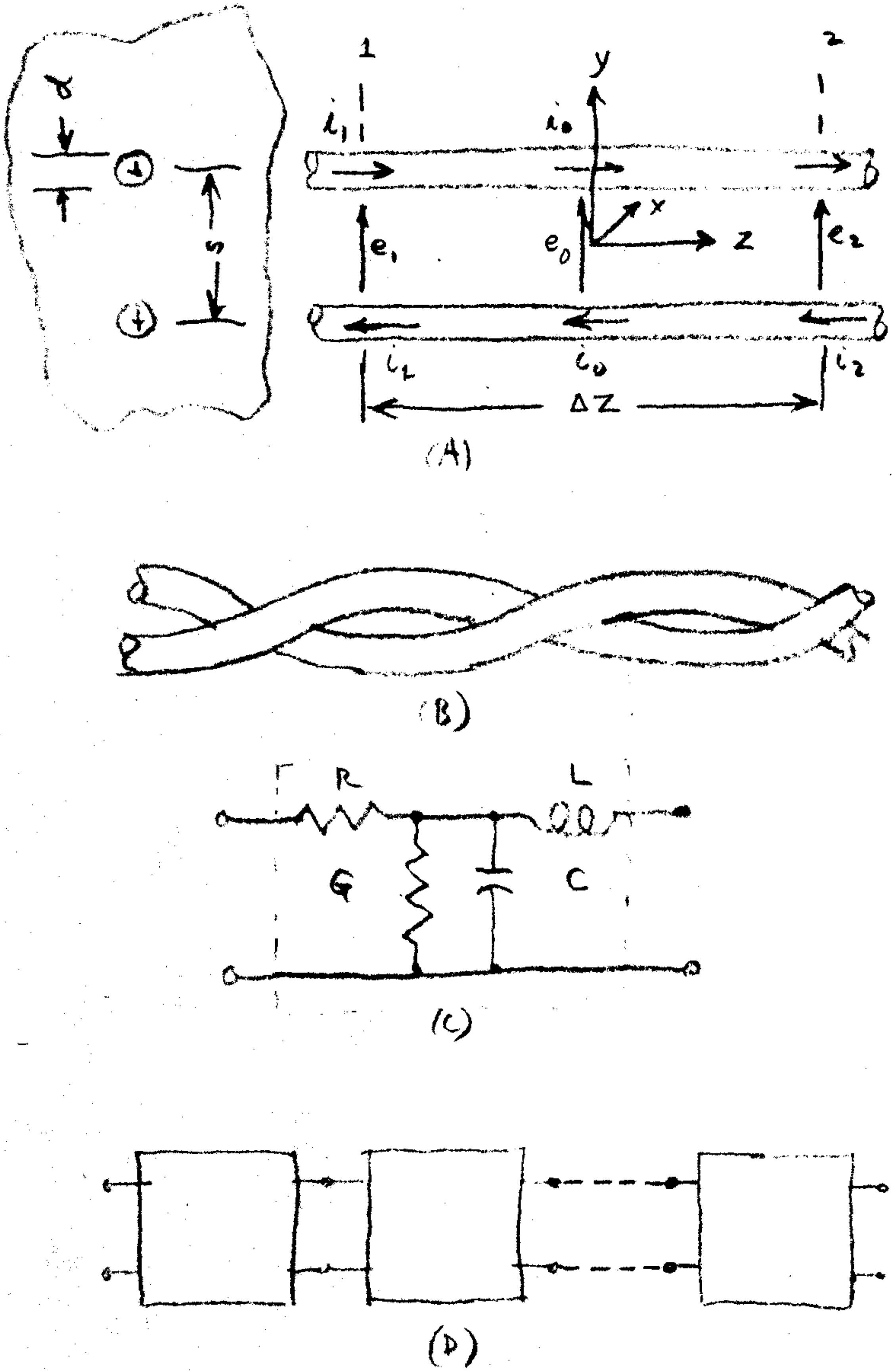

Figure 2. Lumped Constant Representation of the Distributed Constant Transmission Line.

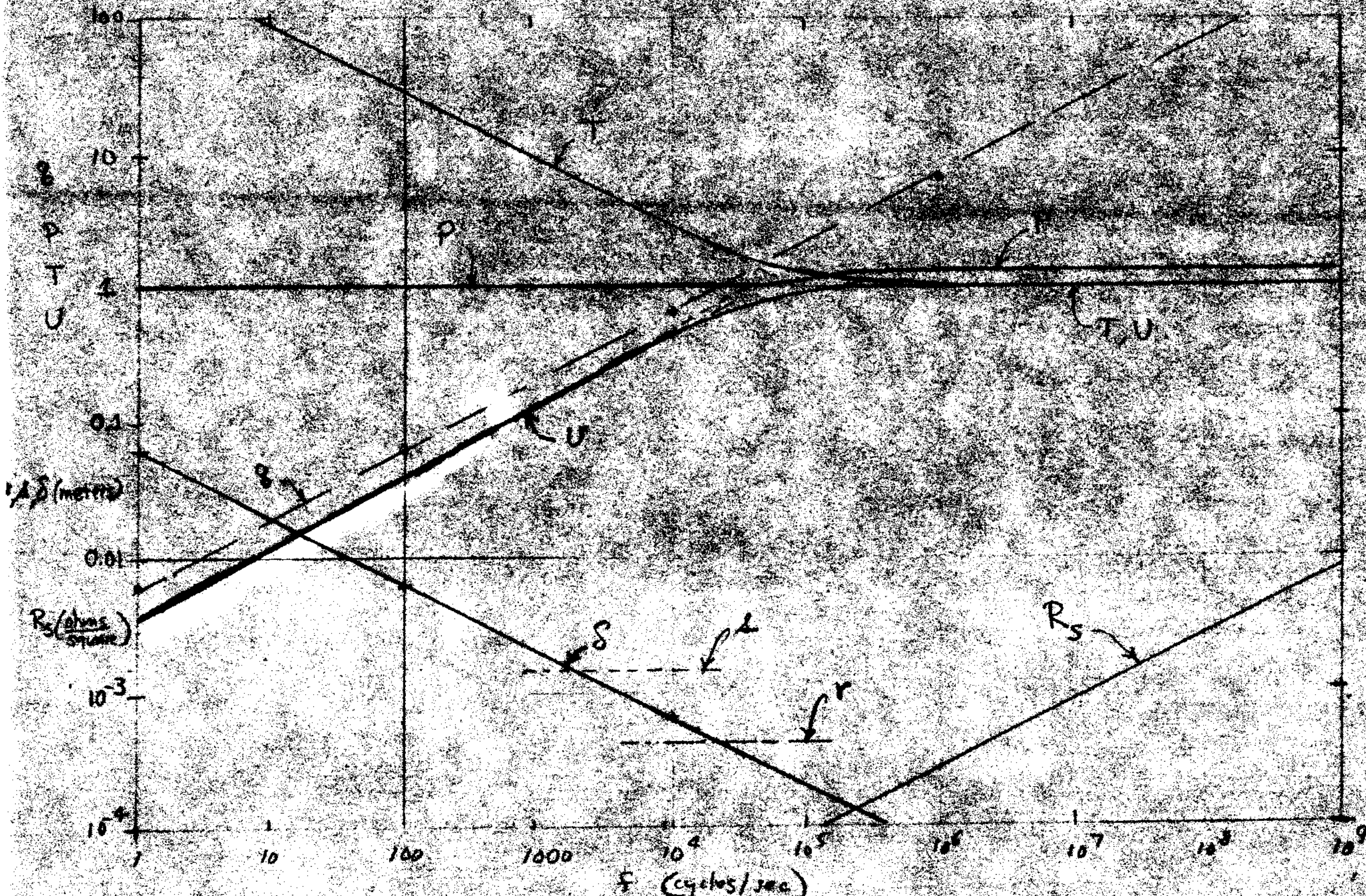

Figure 3 – Skin and Proximity Effect

… (copper)