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Bethe's Lumped Constants

for

Small Irises

by

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Revisions to:

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Since preparation of these notes the following points have been clarified:

Page 5, last paragraph: Bethe rejects the $\bar{K}\phi$ and $(\gamma/\epsilon)\nabla_f\phi$ terms of equations (9) and $\bar{K} \times \nabla_f\phi$ term of (10) in obtaining (15) and (16) on the basis that the differential contributions of these terms fail to satisfy the boundary conditions on the screen. He then, without explanation, doubles the value of K_m and γ_m .

The correct justification for this step is that although some differential elements in equations (9) and (10) fail to satisfy the boundary conditions, the integrated values (i.e. over the whole plane) satisfy the boundary conditions so that these terms cannot be discarded on the boundary condition argument. However in this problem the integrated contribution of \bar{K} , γ over the whole screen (including hole) equals the contribution of \bar{K} , γ_m over the plane (zero except in hole), hence the \bar{K} , γ terms can be replaced by equivalent K_m , γ_m terms, which doubles K_m , γ_m in equations (15) and (16).

Page 6, last paragraph: The above revision which is a more fundamental procedure substitutes multiplying $K_m(r')$ by two instead of dividing $F(0)$ by two, yet giving the same final results.

Page 7, second paragraph: The reversing of the sign of one field (on left) is unnecessary provided the logic of correction to page 5 is followed.

Page 7, footnote*: Bourgin¹⁷ in the meantime pointed out that discontinuities in currents and charges around the edge of the hole were not accounted for. Bouwkamp³ specified this condition mathematically in the form of equation (23).

Page 33A: "Indirect Measurement" means that the point for $d = 0.125"$ was for an iris on the output of a resonator instead of the input as was done with the other sizes. It is planned to make a more reliable measurement of this when the equipment is available again.

Bethe's lumped constants for small irises

The development of the theory of diffraction for small holes and application to irises in waveguides and cavity resonators, published by H. A. Bethe in M. I. T. Radiation Laboratory Reports V-15S(128)^{1A} and 43-22(194)², is here summarized in rationalized U. S. units instead of the original Gaussian unratinalized units. It is not safe to simply transform Bethe's formulas from Gaussian unratinalized units to rationalized U. S. units, because he defines magnetic current density as $\bar{K} = \frac{\bar{E} \times \bar{n}}{2\pi}$ yet there are some equations in 43-22 where the 2π has been dropped and one equation where a 4π required in the unrationalized system is omitted.

For the circular aperture the correct value of the H component of magnetic current given by Bouwkamp³ is included. This error does not, however, change the radiation field and consequently it makes no change in the lumped constant polarizabilities obtained by Bethe.

As in 43-22, the equivalent magnetic and electric polarizabilities are used to obtain the Poynting vector, change of resonant frequency of a cavity, susceptance of an iris in a waveguide, and energy emitted from a cavity through aperture into free space and into a waveguide. These results differ from Bethe's results by functions of 2 and/or 2π due to discrepancies mentioned above. The iris susceptance is compared with other reliable results. The frequency shift and Q_c (coupled Q) for coupling to TE₁₀ waveguide from TM₀₂₀ cavity resonator are compared with experimental data for which the correspondence is fair. (The experimental data is from Cavity measurements which were not designed to give a direct test of Bethe's theory.)

An example of emission through an iris into free space is calculated numerically, which shows that Bethe's statement "that this power is about 25 times greater for emission into free space than for emission into a waveguide of customary dimensions" is not valid. No experimental data has been obtained to check this point.

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1. The Diffraction Problem.

The diffraction of electromagnetic waves by a small hole in an infinite plane conducting screen is studied first, since it is a simpler type of the problem of coupling from a resonant cavity to waveguide.

In the usual Kirchhoff method, the diffracted field is expressed in terms of the incident field in a simple, which, however, does not satisfy the boundary condition. A modified way was later set up a vector modification of Kirchhoff's method,⁵ but this still does not satisfy the boundary conditions when the integrals are taken over the aperture only.⁶ There does exist a rigorous solution by Sommerfeld⁷ for the diffraction of a semi-infinite wedge, which contains no singularities and which is the plausibility of diffraction. T. J. Lowry⁸ has obtained a similar solution for a semi-infinite wedge, for which the correctness of Bell's is the question.

In the literature, the currents H_{00y} and I_0 are all given as the components showing up in the electric field, or in effect with no missing field, so that they are called "physical" in both cases, when I_0 and H_{00y} are the same components shown in the left and right.

4. Interference of light hole's field

Let us consider



Consider an incident wave function

incident on the hole and satisfies, $y = u$

where y is the vertical position, u the

$$u = \sqrt{2k} \sin(ky - \omega t)$$

the phase of the wave function

then the scattered wave function is

$\psi(r) = \frac{1}{r} \left(u_0 + u_1 e^{ikr} \right)$

Then $U_p(r) = \frac{1}{r^2} \left(1 + \frac{2u_0}{r} e^{ikr} \right)$ is the field

at distance r from the hole. If r is large

$$(4) \quad U_p(r) = \frac{1}{r^2} \left\{ 1 + \frac{2u_0}{r} e^{ikr} \right\} \phi(r) + u_0 e^{ikr} \quad \text{for large } r$$

assuming $\phi(r) \ll 1$ means the field at the surface

$r \approx 0$, but $u_0 \ll 1$ is hole to screen. So it's better not to

assume $u_0 \ll 1$. For small hole and $u_0 \ll 1$, $\frac{\partial u_0}{\partial r} \ll 1$ on the hole.

Then,

$$(5) \quad \frac{\partial u_0}{\partial r} \ll 1 \Rightarrow \frac{1}{r^2} \left[\frac{2}{r} \frac{\partial u_0}{\partial r} \phi(r) + u_0 \frac{\partial \phi(r)}{\partial r} \right]$$

for small hole and large r .

There are two different assumptions:

(a) $\frac{\partial u_0}{\partial r} = 0$ near the hole,

then $\frac{\partial u_0}{\partial r} \ll 0$ in

the hole, $\frac{\partial \phi(r)}{\partial r} \ll 0$ for $r \gg r'$
on screen.

(b) $\frac{\partial u_0}{\partial r} = 0$ near the hole,

then $\frac{\partial u_0}{\partial r} \ll 0$, $\frac{\partial \phi(r)}{\partial r} = 2 \frac{\partial u_0}{\partial r}$

hole $\frac{\partial \phi(r)}{\partial r} \ll 0$ for $r \gg r'$
on screen.

So on screen at large r :

$$(a) M_p(r) = -\frac{A}{4\pi} \frac{\partial u_0}{\partial z}, \phi(r) \neq 0 \quad (b) M_p(r) = -\frac{A}{2\pi} \frac{\partial u_0}{\partial z}, \phi(r) \neq 0$$

Since $M = E_z$, a component of $\vec{E} \times \vec{n}$, the boundary condition $\vec{E} \times \vec{n} = 0$ is violated.

Next we consider the vector equivalent of Kirchhoff's Theory by the direct integration of Maxwell's equation using the vector analogue of Green's Theorem as given by Stratton⁹ or by Sjöberg¹⁰.

Maxwell's equations and equations of continuity for \vec{E} (cont'd)

$$(a) \nabla \times \vec{E} + j_0 \mu_0 H = -\vec{J}_m \quad (b) \nabla \cdot \vec{H} + \rho_0 e_0 \epsilon_0 \vec{E} = \vec{J}$$

$$(c) \nabla \cdot \vec{H} = \rho_0 j_m \quad (d) \nabla \cdot \vec{E} = \rho_0 / \epsilon_0$$

$$(e) \nabla \cdot \vec{J}_m + j_0 \epsilon_0 \vec{E} = 0 \quad (f) \nabla \cdot \vec{J} + \rho_0 \epsilon_0^2 \vec{E} = 0$$

Using equations (103, 109, 111) in Maxwell's theory we have at point 'r':

$$(a) \vec{E}_{ext} = \left(-\frac{1}{4\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \mu_0 \epsilon_0 \langle j_m \cdot \vec{J}_m \rangle \delta(\vec{r} - \vec{r}_m) \cdot (\vec{n} \cdot \vec{E}) \right) \hat{z}$$

$$(b) \vec{H}_{ext} = \left(-\frac{1}{4\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \langle j_m \cdot \vec{J}_m \rangle \delta(\vec{r} - \vec{r}_m) \cdot \frac{P_m}{\rho_0} \hat{z} \right) \hat{z}$$

$$(c) \vec{J} = \left(-\frac{1}{4\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \epsilon_0 \vec{n} \times \vec{H}_m \right) \delta(\vec{r} - \vec{r}_m) + \vec{J}_s = \vec{J}_s$$

The electric and magnetic currents in terms of surface charges are expressed

$$(d) \begin{cases} \vec{E} = \vec{n} \times \vec{H} \\ \vec{J}_s = \vec{E} \times \vec{n} \end{cases} \quad \text{where } \vec{n} = \vec{r} / |\vec{r}|$$

The corresponding values in Cartesian axes are as follows:

$$E_x = \frac{\partial \vec{E}}{\partial r} \cdot \vec{e}_x$$

$$E_y = \frac{\partial \vec{E}}{\partial r} \cdot \vec{e}_y$$

$$E_z = \frac{\partial \vec{E}}{\partial r} \cdot \vec{e}_z$$

$$J_x = \frac{\partial \vec{J}_s}{\partial r} \cdot \vec{e}_x$$

$$\vec{F}$$

Since in this problem all source are in the line of the screen, (say) $r_1 = r_2 = R$

$$E(\phi) = \lim_{\epsilon \rightarrow 0} \int_{S^1} (1 - e^{-\epsilon}) \left(1 + \frac{1}{2} \times \mathbb{R}_+^{1+3} \times \mathbb{R}_+^{1+3} \right) \phi$$

$$H(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T E[\hat{S}_T - S_T]^2 d\tau = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T E[\hat{S}_T^2] - 2E[\hat{S}_T S_T] + E[S_T^2] d\tau$$

Equations (2.16) are now solved against the complete boundary condition given by the "vector" equation (2.16). It is found that (2.16) is satisfied at the left boundary and that the right boundary condition is also satisfied. The boundary conditions are therefore satisfied.

3. Mathematical formulation and boundary conditions.

Let \tilde{H}_0 and \tilde{E}_0 be the standing wave field in the left-hand side of the screen if there is no hole. The first condition at eq. 6 is to am-

$$(11a) \quad \nabla \cdot \tilde{\mathbf{H}}_0 = 0$$

$$(11b) \quad \text{Similarly make } \nabla \cdot \tilde{\mathbf{E}}_0 = 0$$

Both \tilde{H}_0 and \tilde{E}_0 are in general not zero or zero. Next we find the diffracted field on the left by \tilde{H}_1 , and the right by \tilde{E}_1 , so the total field with a hole in screen.

$$(12) \quad \left. \begin{aligned} \tilde{\mathbf{E}} &= \tilde{\mathbf{H}}_0 + \tilde{\mathbf{H}}_1 \\ \tilde{\mathbf{H}} &= \tilde{\mathbf{E}}_0 + \tilde{\mathbf{E}}_1 \end{aligned} \right\} \quad \left. \begin{aligned} \tilde{\mathbf{H}}_1 &= \tilde{\mathbf{H}}_0 - \tilde{\mathbf{E}}_0 \\ \tilde{\mathbf{E}}_1 &= \tilde{\mathbf{E}} - \tilde{\mathbf{E}}_0 \end{aligned} \right\}$$

Combining (11a) or (11b) in (12), the loss of energy due to absorption in the screen

$$(13) \quad \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) = - \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right)$$

$$\begin{aligned} \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) &= - \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) \\ \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) &= - \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) \\ \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) &= - \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) \\ \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) &= - \frac{d}{dz} \left(\frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 H^2 \right) \end{aligned}$$

$$(14) \quad \tan^{-1} \frac{E}{H} = \tan^{-1} \frac{E_0}{H_0} = 0^\circ$$

$$(15) \quad \text{normal to } \text{Normal } \Rightarrow \theta = 0^\circ$$

It is convenient to convert to polar system. Instead of "angle" θ

we'll convert by α and β as shown in figure 3. In diffraction plane

concern, the angle α and β of θ is defined as follows. α is the angle between the incident wave vector \mathbf{k}_0 and the normal to the surface. β is the angle between the scattered wave vector \mathbf{k}_1 and the normal to the surface.

The problem now is to calculate \bar{E}_1 , \bar{H}_2 subject to b.c. (11a) on screen and (14a,14b) in the hole. Rethe states: "These conditions are valid irrespective of size and shape of hole."¹² This statement of Rethe's has been reviewed by Silver¹³ and these conditions (14) are found to correspond to the unique solution of the problem. Since $\frac{1}{2} \bar{H}_{0\text{en}} = \bar{H}_{1\text{en}}$ and $\frac{1}{2} \bar{E}_{0\text{en}} = \bar{E}_{1\text{en}}$ Eq.(14) is equivalent to this statement of Smythe: "When any form of electromagnetic wave strikes a thin plane perfectly conducting sheet of any shape, the normal electric and tangential magnetic fields of the original wave are unperturbed in the apertures."

Instead of assuming \bar{E}_1 , \bar{H}_2 in hole equal to the incident wave as in Moöff method, we shall set up some integral equations to be solved for \bar{K}_1 , η_1 , \bar{K}_m , η_m in the hole. From (11a)(9)(10) we see that \bar{K}_1 , η_1 generate $\bar{E}(r)$, $\bar{H}(r)$ which violate b.c. on screen. From (8)(14a)(14b) $\bar{E} = \frac{1}{2} \bar{n} \times \bar{H}_0 e$, $\eta = \frac{\epsilon}{2} \bar{n} \cdot \bar{E}_{0\text{en}}$ which means \bar{K}_1 and η_1 are already known. Similarly we see that \bar{K}_m , η_m generate $\bar{E}(r)$, $\bar{H}(r)$ which ordinary b.c. on screen and still are unknowns to be determined (Only coefficients to be required by (13e), (13)).

Using only the terms which (14) are yet not fixed in the hole and (11a) satisfy b.c. on screen, we make from (9)(10) \bar{K}_m , η_m . (9)(10) reduce to

$$(15) \quad \bar{E}(r) = \frac{1}{4\pi} \int_S \bar{K}_m \times \nabla_f \phi \, dS$$

$$(16) \quad \bar{H}(r) = -\frac{1}{4\pi} \int_S [\omega \epsilon \bar{K}_m \phi + \frac{\eta_m}{N} \nabla_f \phi] \, dS$$

S is the surface surrounding hole in cavity.

¹² There may be some confusion in this statement since in fact it is given in reference [12] and [13], which I have not yet had a chance to check.

The problem now is to determine the values of \bar{E} , \bar{H} and \bar{A} which satisfy b. c. (14) for \bar{E} , \bar{H} and \bar{A} which satisfy b. c. in hole (14). Putting \bar{E} , \bar{H} and \bar{A} in terms of vector and scalar potentials as is done by Sche kuno¹⁶

$$(1) \quad \bar{E} = (-j\omega\mu\bar{A} - \nabla V) + (\nabla \times \bar{F})$$

$$(1) \quad \bar{H} = (\nabla \times \bar{A}) - \nabla U + j\omega\epsilon\bar{F}$$

$$(1) \quad (a) \quad \bar{A} = \int_s \frac{J e^{-jk'r}}{4\pi r'} dS \quad (b) \quad V = \int_v \frac{q_0 e^{-jk'r}}{4\pi\epsilon r'} dv$$

$$(c) \quad \bar{F} = \int_s \frac{\bar{M} e^{-jk'r}}{4\pi r'} dS \quad (d) \quad U = \int_v \frac{m_0 e^{-jk'r}}{4\pi\mu r'} dv$$

The exclusion of terms already fixed in hole and not satisfying b. c. on screen as in (15)(16) gives from (1): $\bar{M} \cdot S = \bar{k}_m$, $dS = d\sigma$; $m_0 + \gamma_m$

$$(18) \quad \bar{E}(r) = -\nabla \times \bar{F}$$

$$(19) \quad \bar{H}_{ext} = -\nabla U - j\omega\epsilon\bar{F}$$

$$(20) \quad \bar{F}(r) = \int_s \frac{\bar{k}_m(r') e^{-jk|r-r'|}}{4\pi|r-r'|} dS(r')$$

$$(21) \quad U(r) = \int_s \frac{m_0(r') e^{-jk|r-r'|}}{4\pi\mu|r-r'|} dS(r')$$

equations (20)(21) are the integral equations to be solved for \bar{k}_m , γ_m where $\bar{F}(r)$ and $U(r)$ are fixed by b.c. (14). A rigorous interpretation of (20) and (21) indicates that the integral is to be taken over the two boundary surfaces separated by the infinitesimal gap S_{ext} . By b.c. (13e) (13) and opposite direction of normal on left and right: $\bar{k}_m(0^-) = -\bar{k}_m(0^+)$; $\gamma_m(0^-) = -\gamma_m(0^+)$.

This makes $\bar{F}(0) = 0$, $J(0) = 0$ from (20)(21), yet by b.c. (14) we know $V(0)$ and $U(0)$ are generally non-zero. This difficulty is resolved by taking half of $V(0)$ and $U(0)$ and integrating over the right surface only i.e. solving for the diffracted wave to the right:

$$(20a) \quad \frac{1}{2} \bar{F}(0) = \int_s \frac{\bar{k}_m(r') e^{-jk|r-r'|}}{4\pi|r-r'|} dS(r')$$

$$(21a) \frac{1}{2} U(r) = \int_{-\infty}^{\infty} \frac{\gamma_m e^{-jkr-rt}}{2\pi} S(k) dk$$

This can be put in the form:

$$F(r, \omega) = \int_{-\infty}^{\infty} \frac{\bar{E} \times \bar{n}}{2\pi} \frac{e^{-jkr+rt}}{(r-k)^2} dS(k)$$

Bethe (1) uses this form (not used in this paper) and (2) uses $\bar{K}_m = \frac{\bar{E} \times \bar{n}}{2\pi}$ and $\gamma_m = \frac{\bar{H} \cdot \bar{n}}{2\pi}$ in gaussian units. In gaussian units $\Im w \bar{H} = 4\pi \mu_0 = 4\pi \frac{\gamma_m}{\delta}$ so the above definition of γ_m can be derived by reversing the sign of the fields on the left which gives $\Im w \bar{H} = \frac{2 \bar{H} \cdot \bar{n}}{\delta}$ so $\gamma_m = \frac{\bar{H} \cdot \bar{n}}{2\pi}$

This means that γ_m in Bethe's definition includes the "magnetic charges" associated with both the diffracted waves to the left and to the right. This means that Bethe's development and this paper give consistent values of $\bar{E} \times \bar{n}$ and $\bar{H} \cdot \bar{n}$, yet Bethe's equations give twice the values of \bar{K}_m and γ_m defined in this paper.

$\bar{n} \cdot$ into (6a) gives: $\nabla \cdot (\bar{E} \times \bar{n}) = -jw \mu_0 \bar{H} \cdot \bar{n}$ or

$$(22) \quad \nabla \cdot \bar{K}_m = \frac{\partial \bar{K}_m}{\partial x} + \frac{\partial \bar{K}_m}{\partial y} \quad \nabla \cdot \bar{K}_m = -jw \gamma_m$$

For an infinite plane wave at an infinite plane screen \bar{H}_0 is constant over the hole. In actual practice and particularly for waveguides the standing wave \bar{H}_0 , \bar{E}_0 will vary across the aperture. When $ka \ll 1$, where a is the farthest point of the aperture edge from center of gravity, taking \bar{H}_0 as constant is still a good approximation for simplifying $U(r, \omega)$ for use in (21a) to solve for γ_m . Caution must be observed to avoid neglecting the curl \bar{H} so that no important components of \bar{E} are lost.



On the contour r of the hole the b.c. $\bar{E} \times \bar{n} = 0$

on the screen requires E_{tan} vanish on the contour

$$(23)* \quad \bar{E}_c = \bar{E} \cdot \bar{n} = 0 \text{ on contour}$$

*This condition was not included in Bethe's original paper, but was published by Bethe in an M.I.T. R.L. report which remained classified for some time. Bourgin¹⁷ in the meantime (published this condition) said some condition like this was necessary.

The magnetic current perpendicular to contour must likewise vanish.

$$(24) \quad \bar{K}_{mp} = \bar{E}_c \times \bar{n} = \bar{E} \cdot (\bar{\alpha} \times \bar{n}) = -\bar{n} \times (\bar{E} \cdot \bar{\alpha}) = 0, K_{mp} = 0 \text{ in C}$$

From (23) we have: $\oint \bar{E} \cdot d\bar{\alpha} = 0 \quad \dots \quad (25a)$

By Stoke's Law: $\oint \bar{E} \cdot d\bar{\alpha} = \int_A (\nabla \times \bar{E}) \cdot \bar{n} d\alpha$

From (6)(2)(25a) $0 = \int \bar{E} \cdot d\bar{\alpha} = -j\omega \mu_0 \int_A \bar{H} \cdot \bar{n} d\alpha = -j\omega \int_H d\alpha$

$$(25b) \quad \int_H d\alpha = 0$$

The total "magnetic charge" or the window is zero.

4. Separation into H and E Components.

Examination of (22) indicates division of fields into two components is possible. $\bar{K}_m = \bar{K}_{mh} + K_{me}$

$$\text{div } \bar{K}_H = -j\omega \gamma_m \neq 0 \quad \text{curl } \bar{K}_H \approx \hat{B} a \cos \theta \text{ curl } \bar{K}_E \approx 0$$

$$\text{div } \bar{K}_E = 0 \quad \text{curl } \bar{K}_E \neq 0$$

$$* B < \theta \leq 90^\circ, B \approx 2a$$

θ is angle of incidence (to normal of path of incident wave; $k_a \ll 1$)

Using " \sim " to mean "order of magnitude of" where $k_a \ll 1$:

$$(20a) \frac{1}{2} \bar{F} = \int_{\text{air}} \frac{\bar{K}_m(r) dS(r)}{4\pi |r-r'|} \sim K_m a \quad (21a) \frac{1}{2} \bar{U} = \int_{\text{air}} \frac{\gamma_m(r) dS(r)}{4\pi \mu |r-r'|} \sim \gamma_m a$$

--- (20a) --- (21a)

Considering the two components:

(i) H component

$$\nabla \cdot \bar{K}_m \sim \hat{B} a \sim \omega \gamma_m \quad (27a)$$

$$K_m \sim \omega a \gamma_m \quad (28a)$$

$$K_m \sim k_a \sqrt{\frac{\mu}{\epsilon}} (\hat{H} \cdot \hat{n})$$

$$E_{mh}(r) \ll \sqrt{\frac{\mu}{\epsilon}} H_m(r)$$

Electric dipoles neglected ($B < \theta \leq 90^\circ$)

$$H_m \sim I (1 + k_a^2)^{-\frac{1}{2}} \frac{\gamma_m}{a}$$

$$H_m = -\nabla U \quad (23a)$$

$$E_m \ll \sqrt{\frac{\mu}{\epsilon}} H_m$$

Normal H_m predominates, which is generated by incident tangential H_{in} .

$$\gamma_m \sim \sqrt{\mu} b \quad (30a)$$

(ii) E component

$$\nabla \cdot \bar{K}_E = 0 \quad (27b)$$

$$\gamma_m = 0 \quad (28b)$$

$$(\hat{H} \cdot \hat{n}) = 0$$

$$H_a(r) = 0 \quad E_{tan}(r) \text{ is present}$$

Magnetic dipoles not present

$$\nabla U = j \omega \epsilon \bar{F}$$

$$U = 0; \quad H_m \sim k_a \sqrt{\frac{\mu}{\epsilon}} K_m$$

$$E_m = -\nabla \times \bar{F} \sim \hat{E} a \sim K_m \quad (23b)$$

$$E_m = -\nabla \times \bar{F} \sim \hat{E} a \sim K_m$$

$$\sqrt{\frac{\mu}{\epsilon}} H_m \ll E_m$$

Tangential E_m predominates, which is generated by incident normal E_{in} .

$$K_m \sim E_0 \quad (30b)$$

5. Solution for the H component.

We neglect terms of order ka since in this approximation $ka \ll 1$.
For uniform incident field by b.c. (14a) and (29a) we have at $z = 0$:

$$(31) \quad \bar{H} = \frac{1}{2} \bar{H}_{oy} = -\nabla U = -\bar{a}_y \frac{\partial U}{\partial x} \quad J = - \int \frac{1}{2} \bar{H}_{oy} dz = -\frac{1}{2} \bar{H}_{oy} = -\frac{1}{2} \bar{H}_0 \bar{r}$$

(31) in (26b) gives integral equation for η_m :

$$(32) \quad -\frac{1}{4} \bar{H}_0 \cdot \bar{r} = \int_{S_0} \frac{\eta_m(r')}{4\pi\mu_0 |r-r'|} dS(r')$$

The magnetic moment is:

$$(33) \quad \bar{X} = \int_{S_0} \eta_m(r') \bar{F}' dS(r')$$

$$\bar{r}' = \bar{a}_x x' + \bar{a}_y y'$$

Using ~~$F' = \bar{a}_x \bar{x}' + \bar{a}_y \bar{y}'$~~ and 22) in (33).

$$\begin{aligned} \bar{X} &= \bar{a}_x X_x + \bar{a}_y X_y = \bar{a}_x \int_{S_0} \eta_m(r') \bar{x}' dS + \bar{a}_y \int_{S_0} \eta_m(r') \bar{y}' dS \\ -j\omega X_x &= \int_{S_0} \times (\nabla \cdot \bar{R}_m) dS, \quad -j\omega X_y = \int_{S_0} \bar{y} (\nabla \cdot \bar{R}_m) dS \\ -j\omega X_x &= \int \times (\nabla \cdot \bar{R}_m) dS = \int \nabla \cdot (\bar{x} \bar{R}_m) dS - \int \bar{R}_m (\nabla \times) dS \end{aligned} \quad (33)$$

1st term by Gauss Law and (2+1): $\int_{S_0} \nabla \cdot (\bar{x} \bar{R}_m) dS = \oint (\bar{x} \bar{R}_m) \cdot \bar{n} dA = 0$

$$\therefore j\omega \bar{X} = \int \bar{R}_m dS \quad \dots (34) \quad \text{or} \quad j\omega \bar{X} = -\bar{a}_x \int \bar{E} dS \quad (34)$$

By (34) \bar{X} is a magnetic dipole moment in the plane of the screen.

It is known, if E_{tan} over aperture is known. (As will be shown in the next section, the electric dipole and hence the complete solution determined by E_{tan} over the aperture.)

Examination of (32)(33) shows that η_m and \bar{X} are linear functions of H_{ox} and H_{oy} , so we write \bar{X} in terms of \bar{M} , the components of a magnetic polarizability tensor.

$$(3) \quad X_x = \vec{M}_{xx} H_{ox} + \vec{M}_{xy} H_{oy}$$

$$X_y = \vec{M}_{yx} H_{ox} + \vec{M}_{yy} H_{oy}$$

The originally incorrect sign of M was corrected by Bethe¹⁸. The sign can be found from the following development related to Stratton's discussion of Poynting's Theorem¹⁹. The stored energy is:

$$(38a) \quad W = \frac{1}{2} \int [E \cdot \vec{D} - \vec{H} \cdot \vec{B}] dv \quad / \text{inst}$$

In the equations marked "inst" the field vector E, D, H, B are the instantaneous real values. In general in this paper, the field vectors indicate the peak values (or the field vector with the time factor $e^{j\omega t}$ omitted). Eq (38a) is also valid where E, D, H, B are average values.

Substituting (18)(19)(6a)(6c) into (38a)

$$(39a) \quad W_{av} = \frac{1}{2} \int \left\{ -e^{-t} (\vec{F} \times \vec{E}) + \mu_0 \epsilon_0 \vec{H} \cdot \vec{F} + e^t \vec{E} \cdot \vec{B} \right\} dv + \frac{1}{2} \int \left\{ \mu_0 \epsilon_0 \vec{H} \cdot \vec{F} + \vec{U}_{ph} \right\} dv$$

$$(39b) \quad \frac{1}{2} \int W_{av} = \frac{1}{2} \int \left(\epsilon \vec{J}_m \cdot \vec{F} + \vec{U}_{ph} \right) dv + \frac{1}{4} \int (\vec{F} \cdot \vec{E} \times \vec{H}) dv$$

The second $1/2$ comes from division at $t = 0$ as in (20a)(21a). The volume is bounded by a hemisphere on the right and the conducting surface of the curved plus \perp surface through aperture. Since the waves are propagated by a finite velocity, we may take the radius of the hemisphere sufficiently large so that the diffraction wave has not reached the sum yet. Then the second integral is zero over the hemisphere. The surface integral over the aperture is zero since $\vec{F} \times \vec{E} = 0$ and $\vec{U}_{ph} = 0$ on the aperture. This leaves

$$(39c) \quad \frac{1}{2} \int W_{av} = \frac{1}{2} \int (\epsilon \vec{J}_m \cdot \vec{F} + \vec{U}_{ph}) dv \quad \text{approx}$$

$$W_{av} = \frac{1}{2} \int \left(\epsilon \vec{J}_m \cdot \vec{F} + \vec{U}_{ph} \right) dv$$

change from instantaneous $\hat{T}_m, \hat{F}, \hat{V}, \hat{J}_m$ to weak values.

$$(40) \quad \frac{1}{2}W = \frac{1}{8} \int_S [U\gamma_m + \epsilon \bar{F} \cdot \bar{K}_m] dS$$

For H component by (29a) second term is of order $(ka)^2$, $ka \ll 1$ so by (21a)

$$(40a) \quad W \approx \frac{1}{8} \int_S U\gamma_m dS = \frac{1}{8\pi\mu_0} \left\{ \left\{ \frac{\gamma_{m(r)} \gamma_m(r)}{|r-r'|} dS(r) dS(r') \right\} \right\} > 0$$

(40a) is positive since it is a self potential. Using (40a)(32)(33):

$$\text{So } \bar{H}_o \text{ and } \bar{\chi} \text{ have opposite signs} \quad \bar{W}_m = -\frac{1}{8\pi\mu_0} \int_S \bar{H}_o \cdot \bar{F} \gamma_m dS = -\frac{\bar{H}_o}{8\pi\mu_0} \cdot \int_S \gamma_m dS = -\frac{\bar{H}_o \cdot \bar{\chi}}{8\pi\mu_0} > 0$$

$$(41) \quad W_m = -\frac{1}{8\pi\mu_0} \bar{H}_o \cdot \bar{\chi} > 0$$

Let $M_{ij} = -M_{ji}$ in (37) then: $M_{ij} > 0$

$$(41a) \quad \begin{cases} \bar{\chi}_x = -M_{xx} H_{ox} - M_{xy} H_{oy} \\ \bar{\chi}_y = -M_{yx} H_{ox} - M_{yy} H_{oy} \end{cases}$$

Bethe points out that the energy relation (41) is analogous to that of the ordinary theory of magnetism: "Therefore, just as we can conclude that the tensor of magnetic permeability is symmetrical, so we believe that in our case we can conclude that the M-tensor is symmetrical." 20

$$(42) \quad M_{yx} = M_{xy}$$

We need only assume that $\bar{\chi}$ is the derivative with respect to \bar{H}_o of:

$$(42a) \quad 8W = \frac{1}{2} M_{xx} H_{ox}^2 + \frac{1}{2} (M_{xy} + M_{yx}) H_{ox} H_{oy} + \frac{1}{2} M_{yy} H_{oy}^2$$

If we accept (42) the tensor $\bar{\chi}$ can be transformed to principal axes. "For non-symmetrical apertures the directions of the principal axes must be determined from the integral equation."

Let $\bar{\ell}$ and \bar{m} be unit vectors in the directions of the two principal axes, and H_{ol} , H_{om} be respective components of H_o . Then in terms of principal axes, the magnetic moment is:

$$(43) \quad \bar{\chi}_{\text{(dipole)}} = -M_1 H_{ol} \bar{\ell} - M_2 H_{om} \bar{m}$$

M_1 , M_2 are the principal magnetic polarizabilities in units of permeability times volume ($M \sim \mu \text{a}^3 \text{ Henry-meter}^2$)

An alternate form from (36b) is:

$$(44) \quad \bar{n} \times \int \bar{E} dS = +j \omega (M_1 H_{ol} \bar{\ell} + M_2 H_{om} \bar{m})$$

6. Solution for E component.

For the electric component we have from (22)(2 b):

$$(45) \quad \nabla_m = \frac{i\beta}{\omega} \vec{r} \cdot \vec{E}_m = \frac{i}{\omega} \vec{r} \cdot (\nabla \times \vec{E}) = 0$$

This means that lines of magnetic current must be closed or that \vec{E}_{tan} can be derived from a potential function:

$$(46) \quad \vec{E}_t = -\nabla \phi$$

(45) and (46) are true irrespective of size of hole. By b.c. (23):

$\phi = \text{constant}$ on the contour of aperture. The constant may be taken as zero without changing E_0 from (46): $\phi = 0 \quad \text{on contour}$ (46a)

$$\begin{aligned} (46)(46a) \quad -\nabla \times \vec{F} &= \frac{i}{2} \vec{E}_{on} & -(\nabla \times \vec{F}) \times \vec{r} &= \frac{i}{2} \vec{E}_{on} \times \vec{r} \\ (\nabla \times \vec{F}) \times \vec{r} &= \vec{F} (\nabla \cdot \vec{F}) - \vec{r} (\nabla \cdot \vec{F}) & \nabla \cdot \vec{F} = 0, \nabla \cdot \vec{r} = 2 \quad (\text{eq. 17 and 18}) \\ -\vec{F} &= \frac{i}{2} \vec{E}_{on} \times \vec{r} & \vec{F} &= -\frac{i}{4} \vec{E}_{on} \times \vec{r} \end{aligned} \quad (46b)$$

Now (46b) and (23a)(23b) give the \vec{E} -field equation to be solved - \vec{E}_{on}

$$-\frac{i}{2} \vec{E}_{on} \times \vec{r} = \sqrt{\frac{K_m(0)}{2\pi \epsilon_0 r^2}} \cos(\theta) \quad (46c)$$

Given $K_m(0) = 0$ on the outer boundary of the cylinder, we may choose the \vec{E}_{on} to be a simple exponential wavelet vector. This vector is given by the components of angular velocity ω in cylindrical coordinates. This restriction is due to the fact of azimuthal symmetry. In view of $\vec{V} \sin \theta = \vec{r} \times \vec{V}$, the \vec{E}_{on} will be $\vec{E}_{on} = \vec{V} \sin \theta$. Then the currents in the two "electrodes" (currents) will be $I_1 = \frac{1}{2} \pi r^2 V \sin \theta$ and $I_2 = -\frac{1}{2} \pi r^2 V \sin \theta$.

$$(47a) \quad \vec{E}_{on} = \frac{V \sin \theta}{2} \hat{z} \quad I_1 - M_1 = -\frac{X_1}{N}$$

The dipole magnetization \bar{X} is used here and everywhere in this paper instead of loop magnetization \bar{M} to make the inhomogeneous wave equations of identical form for both magnetism and electric currents.

$$(47b) \quad \bar{X}_0 = \bar{\chi}_0 = -\frac{i}{\omega} \frac{3m}{4\pi} \hat{r} \times \frac{-i}{\omega} \frac{3m}{4\pi} \hat{r}$$

corresponding to iteration eq. (4)

$$(47c) \quad \bar{\Pi}^* = \frac{e^{i\omega t}}{4\pi} \int \bar{X}_0(\xi) e^{-i\omega|\xi|} d\xi$$

The total field for $\bar{\Pi}^* = \sum_{n=0}^{\infty} \bar{\Pi}^{*(n)}$ is magnetic at $r = 0$ represented by a sum over $\bar{\Pi}^{(n)}$ as inable quadrupole, (first el. dipole, (dipole of current))

Then substituting \bar{X}_0 for $\bar{\chi}_0$ in leading approximation of iteration p. 232, eq. (40) becomes (47f)

$$(47d) \quad \bar{\Pi}^{(0)} = \frac{i}{4\pi\omega} \frac{\bar{\chi}^{(0)}}{R} e^{-i\omega R} \quad \bar{\chi}^{(0)} = \int_{\text{as}} \bar{X}_0 dS \quad (47e)$$

$$\text{From (47d)} \quad \bar{X}_0 = S \bar{\chi}_0 = -\frac{i}{\omega} \frac{3m}{4\pi} \hat{r} \cdot (\hat{a}_x \bar{E}_y - \hat{a}_y \bar{E}_x) \quad (47f)$$

$$(47g, h) \text{ using (360)}$$

$$\bar{X}^{(0)} = \frac{i}{4\pi\omega} \frac{\bar{\chi}^{(0)}}{R} e^{-i\omega R} \quad \bar{E}_t dS = -(\nabla \phi) dS = -\partial_x \phi dS$$

$$\int \bar{E}_t dS = - \int \phi \bar{n} \times d\bar{s} \quad : \quad \bar{\chi}^{(0)} = 0 \quad (47g)$$

This shows that the dipole approximation is exact for the monopole term.

For the quadrupole approximation we take the dipole moment to

$$\bar{\Pi}^{(1)} = \frac{+ik}{4\pi\omega} \left(\frac{i}{R} - \frac{i}{kR^2} \right) e^{i(\omega t - kR)} \quad \int_{\text{as}} \bar{E}_t dS = 8 \pi R^2 \cos \theta d\theta \quad (48)$$

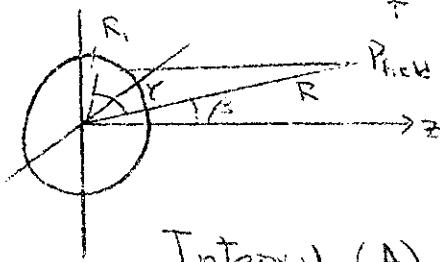
$$\int_{\text{as}} \bar{E}_t dS = 8 \pi R^2 \cos \theta d\theta$$

$$R_i \cdot \bar{R} \cdot \bar{X}_0 = R_i \cdot \bar{R} \cdot \frac{\bar{X}_0}{R} = \frac{1}{2} \bar{R} \cdot (\bar{R} \times \bar{X}_0) \times \bar{R} + \bar{X}_0 (\bar{R} \cdot \bar{R}) + \bar{R} (\bar{R} \cdot \bar{X}_0)$$

$$(48h)$$

Changing to surface integral, the integral part of (48a) becomes.

$$\int_{ns} \tilde{X}_0 R_1 \cos \gamma dS = \underbrace{\int_{ns} \frac{(\vec{R}_1 \times \vec{X}_0) \cdot \vec{R}}{2R} dS}_{(A)} + \underbrace{\int_{ns} \frac{\vec{X}_0 (\vec{R}_1 \cdot \vec{R})}{2R} dS}_{(B)} - \underbrace{\int_{ns} \frac{\vec{R}_1 (\vec{R}_1 \cdot \vec{X}_0)}{2R} dS}_{(C)} \quad (49c)$$



Integral (A) of (49c):

$$\int_{ns} \frac{(\vec{R}_1 \times \vec{X}_0) \cdot \vec{R}}{2R} dS = -\frac{i}{\omega} \int_{ns} \frac{E_x x + E_y y}{2} dx dy \vec{a}_z \times \left(\frac{\vec{R}}{R} \right) =$$

$$\int E_x x dS = - \int \frac{\partial \phi}{\partial x} dS = - \int [\tilde{X}_0 (\phi x) - \phi] dS$$

$$= + \int \phi dS - \tilde{X}_0 (\phi x) \cos(\alpha_r, \vec{x}) dS$$

By (46c) $\int E_x x dS = \int E_y y dS = \int \phi dS \quad \dots \quad (49c)$

Integral (B) of (49c):

$$\int_{ns} \frac{\vec{X}_0 (\vec{R}_1 \cdot \vec{R})}{2R} dS = -\frac{i}{\omega} \int_{ns} \left(\vec{a}_x E_y - \vec{a}_y E_x \right) \left(\frac{x \vec{X}_0 + y \vec{Y}}{2R} \right) dx dy = -\frac{i}{\omega} \left[\vec{a}_x \frac{Y}{R} - \vec{a}_y \frac{X}{R} \right]$$

Integral (C) of (49c):

$$\int_{ns} \frac{\vec{R}_1 (\vec{R}_1 \cdot \vec{R})}{2R} dS = -\frac{i}{\omega} \left(\vec{a}_x \frac{Y}{R} - \vec{a}_y \frac{X}{R} \right) \left(\frac{x \vec{X}_0 + y \vec{Y}}{2R} \right) dS = \frac{i}{\omega} \left[\vec{a}_x \frac{Y}{R} - \vec{a}_y \frac{X}{R} \right]$$

By (49a)(49c) integrals (B) or (C) cancel term (A). Hence $\pi \cdot (49c)$

(49c) $\tilde{\Pi}^{(1)} = \frac{i}{\omega} \frac{q \tilde{X}_0}{2} \int_{ns} \vec{a}_z \cdot dS \cdot \vec{r} \times \nabla R \left(\frac{1}{R} - \frac{1}{R^3} \right) e^{jkr - j\omega t} ; \quad (R = \frac{R}{R})$

Comparing (49c) with Stratonov, p. 404, eq. (21), $\tilde{\Pi}^{(1)} \rightarrow \tilde{\Pi}^{(1)} \text{ eq. (21)}$

We see that $\tilde{\Pi}^{(1)}$ is the contribution of an electric dipole of moment

$$(49d) \quad \tilde{\Pi}^{(1)} = \tilde{Q}_z \in \int_{ns} dS$$

Since the other dipole components there is no remarkable radiation response from the component. $\tilde{\Pi}^{(1)}$ in (49c) or (49d).

$$(50) \quad \bar{P} = \bar{a}_z \in \left\{ \int_{S_1} \bar{E} \cdot \bar{r}' dS \right\}$$

(\bar{E} comes from solution of (46))

In §4 it was shown that the \perp component is generated by the incident normal $\bar{E}(r)$. Consequently we take \perp proportional to normal incident \bar{E} .

$$(51) \quad \bar{P} = \bar{n} P (\bar{E}_0 \cdot \bar{n})$$

P is the electric polarizability of the aperture $P \text{ nes}^3$ (farad-meter³)

Alternately (50) can be obtained from \bar{K}_m directly. Examining

$$\bar{m} = \frac{\bar{X}}{\mu} = \frac{1}{2} \int \bar{E}_i \times \bar{F} dS \quad \text{in Stratton's form and noting sign difference from (6a,b):}$$

$$(50a) \quad \bar{P}_e = \frac{1}{2} \int \bar{K}_m \times \bar{F} dS$$

$$(50b) \quad \bar{P} = \frac{1}{2} \int (\bar{E} \times \bar{n} \times \bar{F}) dS = \frac{\epsilon}{2} \int [\bar{n}(\bar{E} \cdot \bar{F}) - \bar{F}(\bar{E} \cdot \bar{n})] dS = \bar{a}_z + \frac{\epsilon}{2} \int \bar{E} \cdot \bar{r}' dS$$

Note (50b) is same as (50). To determine sign of P in (51) we use (46,47)

$$(52) \quad \frac{1}{2} \bar{W}_{rr} = \int_{S_1} \bar{E} \cdot \bar{K}_m dS$$

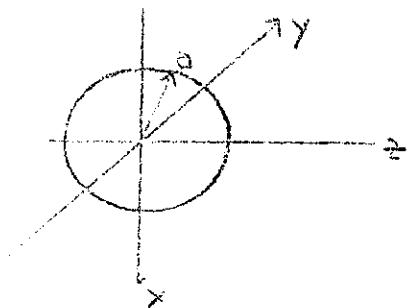
$$(53) \quad \text{From (46)(52)} \quad \frac{1}{2} \bar{W}_{rr} = \frac{\epsilon}{16\pi} \left(\sum_{r' \neq r} \frac{\bar{K}_m(r) \cdot \bar{K}_m(r')}{|r-r'|} dS(r) dS(r') \right) > 0$$

$$\text{From (53)(52)(46)} \quad \frac{1}{2} \bar{W}_{rr} = (-\bar{E}_{on} \times \bar{r}) \cdot \bar{K}_m dS = \frac{\epsilon}{4} \bar{E}_{on} \cdot \int \bar{K}_m \times \bar{r} dS$$

$$\frac{1}{2} \bar{W}_{rr} = \frac{\epsilon}{32} \bar{E}_{on} \cdot \left(\frac{2\bar{P}}{\epsilon} \right) = \frac{\bar{E}_0 \cdot \bar{n}}{16} P (\bar{E}_0 \cdot \bar{n}) = \frac{1}{16} P (\bar{E}_0 \cdot \bar{n})^2 > 0 \quad (54)$$

Since $(\bar{E}_0 \cdot \bar{n})$ is positive, P is positive. The difference in sign between P and \bar{P} is not due to their being magnetic and electric, but due to being parallel respectively to the screen.

Example of Circular Aperture.



Bethe¹ has obtained a solution for a circular aperture of radius a . Bouwkamp³ has corrected Bethe's solution which gave incorrect E_{tan} for H component near aperture and found Bethe's radiation field results to be correct.

Bouwkamp has also obtained a series solution of which the first term corresponds to Bethe's approximation developed here.

a. E Component. (Corresponds to TE mode circular iris of Stratton^{2b})

The integral equation is (32):

$$(35)(32) \quad -\frac{1}{4} \nabla \cdot \vec{H}_0(r) = \int_{\text{boundary}} \frac{J_m(\zeta r)}{\zeta \sin(\zeta a) - \cos(\zeta a)} dS(\zeta)$$

Since Fockamp's correct solution in oblate spheroidal coordinates has not yet been published (to appear in second of two papers), we will outline Rupke's procedure and then give the correct solution given without formal derivation in Bouwkamp's first paper.

It has stated that a compact radial field is produced by a uniform distribution of dipoles in an ellipsoid as is envelope by Stratton^{2b}. Consider the situation to be filled by an ellipsoid of semi-axes a, b, c .

$$\text{where } \frac{c}{a} \cdot \frac{b^2}{a^2} = 1 \quad \Rightarrow \quad (a^2 - r^2)^{1/2}$$

$$\text{Dipole surface density } \Sigma = \sigma \vec{H}_0 \cdot \vec{n} \quad (35)$$

$$\text{Surface charge density } \rho_m = \sigma \vec{X} \cdot \vec{n} = \sigma \vec{E}_0 \cdot \vec{n} \quad (35)$$

$$\int dA \vec{J}_A = \vec{T} \cdot \vec{\rho}_m \vec{n} \quad \text{for } \vec{A} = \vec{G}, \vec{B} \quad (35)$$

$$J_m = \vec{H}_0 \cdot \vec{n} \quad \text{as } \vec{n} \perp \vec{H}_0 \quad (35)$$

$$\sigma \vec{e} \cdot \vec{n} = \frac{\vec{E}_0}{(a^2 - r^2)^{1/2}}, \quad \rho_m = \sigma \vec{E}_0 \cdot \vec{n} = \frac{\vec{E}_0 \cdot \vec{n}}{(a^2 - r^2)^{1/2}} \quad (35)$$

$$H_0 \cdot Y^1 = H_0 Y + \sum_{\alpha} p_{\alpha} \cos(\alpha \theta / \pi) \quad \text{---(64)}$$

$$(56b) \text{ in } (55) -\frac{1}{4} \bar{H}_0 \cdot \vec{F} = \alpha \int_{-\infty}^{2\pi} \frac{\frac{dP}{d\theta}}{4\pi/c} \int_0^P \frac{H_0 r + H_0 P \cos(\theta+\phi)}{(Q^2 - r^2)^{1/2} P} r dr d\theta - (57)$$

1

61

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Integrals I, II are πr^2 , $-\pi r^2$ respectively by Prop. 127; III is zero by Prop. 132.

$$-\frac{1}{4} \overline{H_0} \cdot \vec{F} = \frac{\alpha}{\pi m} \int_0^{\pi} (\overline{H_0} \cdot \vec{F} - d \sigma \cos \cos(\omega n)) dn = \frac{\alpha \pi}{\pi m} (\frac{1}{2} \overline{H_0} \cdot \vec{F})$$

$$x = \frac{3\pi}{4}$$

To obtain the correct value for \tilde{K}_{n_0} , we have to go back to (24). Both Bethe calculated \tilde{K}_{n_0} from (22) without $\tilde{K}_{n_0} \neq 0$ and thereby missed one of \tilde{K}_{n_0} , which caused \tilde{K}_{n_0} to fail in bet.b.c. (24). $\tilde{K}_{n_0} \neq 0$ on curve 1.

Boukamp³ has obtained the following result to the first term in power series in $(k\delta)^{1/2}$, the ideal conditions:

$$(56) \quad K_{\text{ext}} H = -\frac{3\pi}{4} \frac{(2x^2 - 2y^2)}{\sqrt{x^2 - y^2}}$$

True (5%) across trials (1-8).

b. *Alzatea* (Lindau) rank: 100% negative to 100% to circular leaf.

Boukman did not expect that all would consent to his proposal. Since
Bastille Day was the first of many important occasions to come, it might
be wise to postpone the date.

From (46c) the integral equation is:

$$(59)(46c) \quad -\frac{1}{\epsilon} \bar{E}_{\text{ext}} \cdot \bar{r} = \int_{D_0} \frac{\bar{K}_{\text{me}}(r')}{4\pi|r-r'|} dS(r')$$

$$(59a) \quad -\frac{1}{\epsilon} \bar{E}_{\text{ext}} \cdot \bar{r} = \int_{D_0} \frac{\bar{K}_y}{4\pi|r-r'|} dS$$

$$(55) \quad -\gamma_y H_{oy} = \int_{D_0} \frac{M_{oy} dS}{4\pi|r-r'|}$$

Comparing components with (55) we see that they are the same equations with different symbols, so we can use (57d) which gives by comparison:

$$(60) \quad \boxed{\bar{K}_{\text{me}} = \frac{1}{\pi(a^2-r^2)^{1/2}} \bar{r}' \times \bar{E}_0} \quad (\text{volts})$$

c. Magnetic Polarizability.

$$(61) \quad \bar{x} = -\frac{1}{\omega} \int (\bar{K}_{\text{ext}} + \bar{K}_{\text{me}}) dS \quad - \text{from } (56c) \text{ or } (57b)$$

The x-component of \bar{K}_{ext} and the sole \bar{K}_{me} part will both integrate to zero.

It is simpler to use (33) and (57d):

$$\begin{aligned} \bar{x} &= \int_{D_0} (\gamma_y(r) \bar{r}' dS(r)) = \bar{a}_y \int_{D_0} \bar{r}' \times dS + \bar{a}_y \int_{D_0} M_{oy} dS = \bar{a}_y \left(\frac{-2L}{\pi} \right) \left(\frac{r^2}{a^2} \right) \left(\frac{1}{a^2-r^2} \right) \\ (62) \quad \bar{x} &= -\bar{a}_y \frac{4a^3}{3\pi} \frac{1}{a^2-r^2} \end{aligned}$$

$$(62) \quad M_{oy} = -\frac{\bar{x}}{\bar{a}_y} = -\frac{4}{3} a^3 \mu = M_{oy}$$

d. Dielectric Polarization:

$$(63) \quad \bar{p}_z = \frac{1}{\epsilon_0} \int_{D_0} (\bar{r}' \cdot \bar{r} dS) = \frac{1}{\epsilon_0} \int_{D_0} ((\bar{r}' \times \bar{K}_{\text{me}}) \cdot \bar{r} dS) = \frac{1}{\epsilon_0} \int_{D_0} (\bar{r}' \times \bar{r}) \cdot (\bar{r}' \times \bar{r}) dS = \frac{\epsilon_0}{2\pi} \frac{1}{a^2-r^2} = \frac{\epsilon_0}{2} \frac{4}{3} a^3$$

$$(64) \quad \bar{p}_z + \frac{2a^3}{3} \epsilon E_0 = -(\bar{a}_y) \times (51)(61) \quad \bar{p}_z = \frac{\bar{p}}{E_0} = \frac{1}{2} + \frac{2a^3}{3} \epsilon = P$$

8. Summary of Results for Various Types.

a. Circular Aperture. Radius a

Bethe's Gaussian System

$$\bar{R} = \frac{\bar{E} \times \bar{n}}{2\pi} \quad \text{and} \quad \eta_m = \frac{\bar{H} \cdot \bar{n}}{2\pi}$$

$$M'_1 = M'_2 = \frac{4}{3} a^3 \quad \text{--- (7c)*}$$

$$P' = \frac{2}{3} a^3 \quad \text{--- (7d)}$$

$$\bar{x}' = -\frac{M'_1}{2\pi} \bar{H}_0' \text{ (remu)}$$

N.V.S. from §7 this paper:

$$N = N/\rho_0 \quad \rho_0 = 47 \times 10^{-7}$$

$$\frac{M_1}{\rho} = \frac{M_2}{\rho} = \frac{4}{3} a^3 \quad \text{--- (7e)*}$$

$$\frac{P}{\rho} = \frac{2}{3} a^3 \quad \text{--- (7f)}$$

$$\bar{x} = -M_1 H_0 \text{ (nuclear magneton)}$$

* These two different methods of dividing the problem into left and right halves are consistent as far as the solution for $\bar{E} \times \bar{n}$ and $\bar{H} \cdot \bar{n}$ in each aperture. The definition by Bethe of $\eta_m = 2(\frac{\bar{H} \cdot \bar{n}}{4\pi})$ amounts to multiplying each factor of 2 into the magnetic and electric moments.

* From dimensional analysis or analogy we see: $\frac{M}{\rho} = \frac{4\pi}{3} P' = \rho$.

The difference in the two derivations shown above reduces this definition to $(\frac{M}{\rho} = \frac{1}{2} P')$ if the $\frac{1}{2}$ is taken.

Bethe apparently split the problem into two symmetrical parts and had H_{2tsn} be H after (1.1.1) by setting $\eta_1 = \frac{H \cdot \bar{n}}{4\pi}$ but $\eta_{2tsn} = \frac{H \cdot \bar{n}}{4\pi}$. In this paper the equivalent division was made by dividing H into H_1 and H_2 at (2.1.1).

Checking against results for M in (1.1.1) it is found that the corresponding formulae for M are identical excepting $\frac{M}{\rho} = 1$. The results for other quantities are transposed from Bethe's Gaussian function without change. The author has developed no new basis for the division of the current.

b. Circular Aperture.

Bethe regards this as the basic problem, but it is not the only one.

inside an elliptic window, which is to the following:

$a = \text{major axis}$ $b = \text{minor axis}$

eccentricity $E = \sqrt{1 - (\frac{b}{a})^2}$

F, E are complete elliptic integrals; $(84a)$

$$F = F(E) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - E^2 \sin^2 \theta}} \quad E = E(\bar{E}) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 + E^2 \sin^2 \theta}} \quad (84b)$$

$$\frac{M}{N} = \frac{\pi}{3} \frac{a b^2 E^2}{(1 + E)^2 (1 - E)} \quad M = \frac{\pi b^2}{3} \frac{E^2}{(1 + E)^2} \quad N = \frac{\pi}{3} \quad (84c)$$

For small E , $E \approx \frac{\pi}{2} [1 - \frac{1}{2} E^2]$ $(84d)$ $F \approx \frac{\pi}{2} [1 + \frac{1}{2} E^2]$ $(84e)$

from which it can be seen that $(84c)$ reduces to $(80, 81)$ for $a = b$.

For large E ($E \rightarrow 1$) \rightarrow $F \approx 0.444 E^2$ $(84f)$

$$\frac{M}{N} = \frac{\pi}{3} \frac{a^2}{\sin^2 (0.444 E^2)} \quad \frac{1}{N} = \frac{\pi}{3} a^2 E^2 \quad \frac{E}{N} = \frac{\pi}{3} a^2 \quad (84g)$$

a. Approximate form $(87a)$

$$\frac{E}{N} = \frac{\pi}{3} a^2 \quad (87a)$$

b. General form $(87b)$

which is the same as the general formula for the elliptical window.

and $F = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - E^2 \sin^2 \theta}}$ $(84b)$

then the elliptical window condition becomes

$$\frac{E}{N} = \frac{\pi}{3} a^2 = \frac{D}{b} d^2 \quad (87b)$$

where $D = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - D^2 \sin^2 \theta}}$ $(84b)$

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9. The Poynting Vector.

The time average Poynting vector: $S = \frac{1}{2} \operatorname{Re}(\vec{E} \times \vec{H}^*)$ Integrating over

the aperture and using peak values of E , H , since time factors cancel out

$$(90) \quad S = \frac{1}{2} \int_A \vec{E}_1 \times \vec{H}_1 \cdot \hat{n} dS$$

\vec{E}_1, \vec{H}_1 are fields generated in aperture by incident field \vec{E}_0, \vec{H}_0 .

If we resolve the diffraction field in the right into normal modes, then

$$(91) \quad \text{for mode } i, \quad S_i = \frac{1}{2} \int_A \vec{E}_{2i} \times \vec{H}_{2i} \cdot \hat{n} dS$$

$\vec{E}_{2i}, \vec{H}_{2i}$ are fields of the normal mode i to be excited on the right. \vec{E}_{2i} is obtained from the theory of waveguide resonators. We now use the theory obtained from the first part of this paper to get \vec{H}_2 in terms of \vec{E}_0, \vec{H}_0 . We find a general form for \vec{H}_2 in about e.g. in a Taylor series $\vec{H}_2 = \vec{H}_2(0) + (\vec{r} \cdot \nabla) \vec{H}_2 + \dots$. Keeping only first two terms of (91), $S = S_1 + S_2$ where

$$(92) \quad S_1 = \frac{1}{2} \int_A (\vec{E}_1 \times \vec{H}_2(0)) \cdot \hat{n} dS = \int_A (\vec{E}_1 \times \vec{E}_2(0)) \cdot \hat{n} dS = \int_A \omega [M_1 H_0 L H_2(0) + M_2 H_{0x} H_{2x}] dS$$

$$(93) \quad S_2 = \frac{1}{2} \int_A (\hat{n} \times \vec{E}_1) \cdot (\vec{r} \cdot \nabla) \vec{H}_2(0) dS$$

$$\vec{r} \cdot \nabla = x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \quad \text{so} \quad S_2 = \frac{1}{2} \int_A \left[E_{1y} \left(\frac{\partial H_{2x}}{\partial x} + \frac{\partial H_{2y}}{\partial y} \right) - E_{1x} \left(\frac{\partial H_{2y}}{\partial x} + \frac{\partial H_{2x}}{\partial y} \right) \right] dA$$

Part of the terms integrate to zero as in (49).

$$S_2 = \frac{P}{\epsilon} \left(\frac{\partial H_{2x}}{\partial x} + \frac{\partial H_{2y}}{\partial y} \right) \quad \Rightarrow \quad (6.6) \quad S_2 = \frac{1}{2} \operatorname{Re} \vec{E}_{0x} \vec{H}_{2x}$$

$$(12)(94) \quad \left\{ \begin{array}{l} S = S - S_2 = \frac{1}{2} \left[M_1 H_0 L H_2(0) + M_2 H_{0x} H_{2x} + \operatorname{Re} \vec{E}_{0x} \vec{H}_{2x} \right] \end{array} \right.$$

In general E_n and H_{tan} are 90° out of phase so that E_{on}, E_{2n} is negative when H_0, H_2 is positive.

Although Bethe's development is in gaussian units, there is no $\frac{c}{4\pi}$ in his *W* vector, Ref. 2, eq. 51. It is safer to start from the beginning in M. S. units, than to transform Bethe's formulas to V. U.S. units. The factor $\frac{c}{4\pi}$ does appear correctlyⁱⁿ, Ref. 2, eq. 6.

10. Change of Resonant Frequency of Cavity.

From Bethe and Schwinger's² perturbation theory for cavities, the shift in resonant frequency of a resonator due to a small window is:

$$(96) \quad \Delta k = k_1 - k_2 = \frac{\int \bar{E}_1 \times \bar{H}_2' \cdot \bar{m} dS}{\int (\epsilon E_2^2 + H_2'^2) dV} \quad (\text{gaussian})$$

Transforming to M.K.S. units and checking with (3) p. 81^(ref) for consistency:

$$(96a) \quad \omega_1 - \omega_2 = \frac{\frac{1}{2} \int \bar{E}_1 \times \bar{H}_2' \cdot \bar{m} dS}{\frac{1}{2} \int (\epsilon E_2^2 + H_2'^2) dV} \quad (\text{M.K.S.})$$

Although the results calculated from this agree with experimental values, this development should be checked.

$$(96b) \quad \bar{H}_2 = +jH_2 \quad \omega_1 - \omega_2 = \frac{\int \bar{E}_1 \times \bar{H}_2' \cdot \bar{m} dS}{\int (\epsilon E_2^2 + H_2'^2) dV} \quad (\text{96c})$$

$$(97) \quad \frac{\Delta \omega}{\omega} = \frac{\frac{1}{2} [P_{in} E_{2m} + j M_1 H_{2x} + j M_2 H_{2y}]}{\frac{1}{2} \int (\epsilon E_2^2 + H_2'^2) dV}$$

This is for a resonant cavity with a source of energy inside radiating out through the window where no energy is reflected back toward the cavity. Practically it is supposed to be good as long as the external incident wave or reflected wave has $\lambda_{\text{EM}} \ll \frac{W}{\Delta \omega}$. Since $\bar{E}_0 \bar{H}_0$ is the field of the cavity itself and is identical with $\bar{E}_2 \bar{H}_2$, we have:

$$(97a) \quad \frac{\Delta \omega}{\omega} = \frac{P E_{2m}^2 - M_1 H_{2x}^2 - M_2 H_{2y}^2}{\int (\epsilon E_2^2 + H_2'^2) dV}$$

From Bethe and Schwinger² or from SI text²⁶ the increase of volume of a resonant cavity is:

$$(97b) \quad \frac{\Delta \omega}{\omega} = \frac{(\epsilon E^2 - \mu H^2) \Delta V}{\int (\epsilon E^2 + \mu H^2) dV}$$

Comparison with (9 a) shows that $\frac{\rho}{\epsilon} \frac{M_1}{N}, \frac{M_2}{N}$ represent the effective increase in volume due to a window in a cavity.

Sample calculation of $\frac{\Delta\omega}{\omega}$ from experimental data of F. B. Wood.²⁹

Brass TM_{020} resonator. Dimensions are shown in figure on page 331.

$$\frac{M_1}{N} = \frac{\pi l d^2}{16} \quad k_c = \frac{\omega_0^2}{a} = \frac{5.52}{a} \quad \gamma_1 = \sqrt{\frac{N}{\epsilon}}$$

$$E_2 = E_0 J_0(k_c r) \quad H_1 = j \frac{\epsilon_0}{\eta_1} J_1(k_c r)$$

$$\text{stored Energy: } V_E = \int \frac{\epsilon E^2}{2} dr = \pi \epsilon h E_0^2 \frac{a^2}{2} J_0^2(k_c a)$$

$$\text{By (17a)} \quad \frac{\Delta\omega}{\omega} = - \frac{M_1 H_1'}{S(\epsilon E_2^2 + N H_1^2) dv} = - \frac{M_1 |H_1|^2}{2 \int \epsilon E_2^2 dv}$$

$$\frac{\Delta\omega}{\omega} = - \frac{M_1 |H_1|^2}{4 V_E} = - \frac{\left(\frac{\pi N h d^2}{16} \right) \left(\frac{\epsilon_0^2}{\eta_1^2} \right) J_0^2(k_c a)}{4 \pi \epsilon h E_0^2 \frac{a^2}{2} J_0^2(k_c a)}$$

$$\frac{\Delta\omega}{\omega} = - \frac{1}{32} \left(\frac{d}{a} \right)^2$$

d''	da	$(\frac{d}{a})^2$	$-\frac{1}{32} \left(\frac{d}{a} \right)^2$	$\left(\frac{\Delta\omega}{\omega} \right)_{\text{exp.}}$
.125	.109	.0128	-.0004	-.00036
.250	.218	.047	-.0015	-.0014
.500	.435	.19	-.0039	-.0054

a = radius of resonator

a = width of window

II. Susceptance of Iris in Waveguide.

a. General Case:

We consider a waveguide of arbitrary cross-section with metal diaphragm extending across it having a small window. The excitation on the right left is given and it is assumed that there is no strong reflected wave coming back from right. The exciting field on left may be in (1) waveguide of same cross-section, (2) waveguide of different cross-section (3) free space, or (4) a resonant cavity.

From Silver³⁰ or Rothe³¹, the normal modes in the waveguide on the right are:

$$\bar{E}_z = \sum_a A_a \bar{E}_{az} e^{-j k_a z} \quad (18) \quad \bar{E}_x = +j \sum_a A_a E_{am} e^{-j k_a z} \quad (18)$$

$$k_a = 2\pi/\lambda_a \quad (18) \quad H_z = +j \sum_a A_a H_{am} e^{-j k_a z} \quad (18)$$

$$\bar{H}_x = \sum_a A_a \bar{H}_{ax} e^{-j k_a z} \quad (18)$$

The amplitude factor is: $A_a = S_{\text{hole}} / S_{\text{waveguide c.s.}}$

From (18)(19)(25)(33b)(34c):

$$(19) \quad A_a S_a = \frac{1}{2} \int_{\text{hole}} \bar{E}_z \times \bar{H}_{ax} \cdot \hat{n} dS$$

$$(20) \quad S_a = \frac{1}{2} \int_{S_a} (\hat{n} \times \bar{E}_z) \cdot \bar{H}_x dS$$

$$(21) \quad A_a = \frac{j k_a}{2 S_a} [M_1 H_0 e^{H_0} + M_2 H_m H_{mm} + j P E_{az} \bar{E}_z] \quad (21)$$

b. Approximation of same Cross-section on both Sides.

Suppose that for number k_a or k that only one normal mode is concerned on right. Let $k_0 = 1$ for first dominant. E_0 , H_0 odd, while E_0 , H_0 even. Then

$$(22) \quad \bar{H}_x = \bar{H}_{ax} (e^{-jk_0 z} + e^{jk_0 z}) = 2 \bar{H}_{ax} \cos k_0 z$$

$$(23) \quad \bar{E}_z = k \bar{H}_{ax} \quad \text{and} \quad \bar{E}_{az} = +k \bar{E}_{am} \quad \text{at } z = 0$$

$$(102) \text{ in (101): } A_a = -\frac{2jw}{2S_a} [P E_{lm} E_{am} - M_1 H_{al} H_{ac} - M_2 H_{am} H_{am}]$$

$$(103)$$

From (103) we can compute the amplitude of each possible mode on the right.

In this case there is only one, i.e. $a = b$, so:

$$(104) \quad A_a = -\frac{2jw}{2S_a} (P E_{lm}^2 - M_1 H_{al}^2 - M_2 H_{am}^2)$$

The susceptance B (for $\gamma_0 = 1$) of an iris is related to the transmission coefficient by: $A_a = \frac{1}{1 + j \frac{B}{2}}$ (105)

For small A_a : $B \approx -\frac{2j}{A_a}$ (105a)

By (104)(105a):

$$(106) \quad \boxed{\frac{1}{B} = \frac{w}{2S_a} (P E_{lm}^2 - M_1 H_{al}^2 - M_2 H_{am}^2)}$$

c. Iris in \mathbb{H}_{10} waveguide.

$$\gamma^2 = k^2 - \frac{\pi^2}{a^2}$$

$$E_{ay} = A \frac{ik}{\beta} \sin \frac{\pi x}{a}$$

$$H_{ax} = -A \sqrt{\frac{E}{P}} \sin \frac{\pi x}{a}$$

$$H_{az} = +j \frac{\pi}{a} \frac{A}{\beta} \sqrt{\frac{E}{P}} \cos \frac{\pi x}{a}$$

For comparison this corresponds to treatment of Ramo and Whinnery³² when $E = j \frac{\pi}{a} \frac{A}{\beta} \sqrt{\frac{E}{P}}$

$$(107) \quad S_a = \frac{1}{2} \int_{\text{guide}} (\vec{n} \times \vec{E}_c) \cdot \vec{H}_c dS = A^2 \frac{\pi b}{4} \frac{k}{\beta} \sqrt{\frac{E}{P}} \quad (107)$$

By (106)

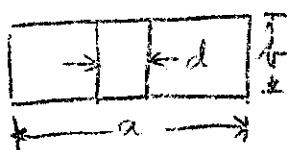
$$(108) \quad B \frac{A}{\beta} = -\frac{a b \lambda_0}{4\pi \left(\frac{M_1}{P}\right) \sin^2 \frac{\pi x}{a}} \quad (108)$$

from $E_{on} = 0$, $H_{al} = H_{ax} \cos(\theta, \hat{x})$, $H_{am} = H_{ax} \sin(\theta, \hat{x})$

$$109 \quad \frac{1}{B} = -\frac{2\pi}{\lambda_0} \frac{1}{\beta} \sin^2 \frac{\pi x_0}{a} [M_1 \cos^2(\theta, \hat{x}) + M_2 \sin^2(\theta, \hat{x})] \quad (109)$$

x_0 = distance to e.g. of slit

For an inductive slot. $X_0 = \frac{a}{d}$ by (88) in (108)



$$\frac{B}{V_0} \frac{a}{\lambda_g} = - \frac{4}{\pi^2} \left(\frac{1}{d} \right)^2 \quad (108n)$$

From the Waveguide Handbook³³ we obtain a more precise solution for comparison (derived in gaussian units, but requires no change for M.K.S. since it is a dimensionless ratio)

$$\left| \frac{V_0}{B} \frac{\lambda_g}{a} \right| = \tan^2 \frac{\pi d}{2a} \left(1 + \frac{1}{4} \sin^2 \frac{\pi d}{a} \left(\frac{1}{\sqrt{1 - (\frac{2d}{\Delta})^2}} - 1 \right) \right. \\ \left. + 2 \left(\frac{2d}{\Delta} \right)^2 \left[\frac{E(\Delta) - \Delta F(\Delta)}{\pi} \frac{E(\Delta) - \Delta^2 F(\Delta)}{\Delta^2} - \frac{1}{\Delta} \sin^{-1} \frac{2d}{\Delta} \right] \right)$$

$$\Delta = \sin \frac{\pi d}{2a} \quad \Delta' = 4 \frac{\pi d}{2a} \quad E, F \text{ are elliptic integrals} \quad (108t)$$

$$\text{For small } \frac{d}{a}: \left| \frac{2d}{a} \right| \ll \frac{\pi d}{2a} \rightarrow \frac{\pi d}{2a} \left(\frac{d}{a} \right)^2$$

Comparing (108t) with (108n), we see they are the same. To make one plotted on reflection loss page 3, note that

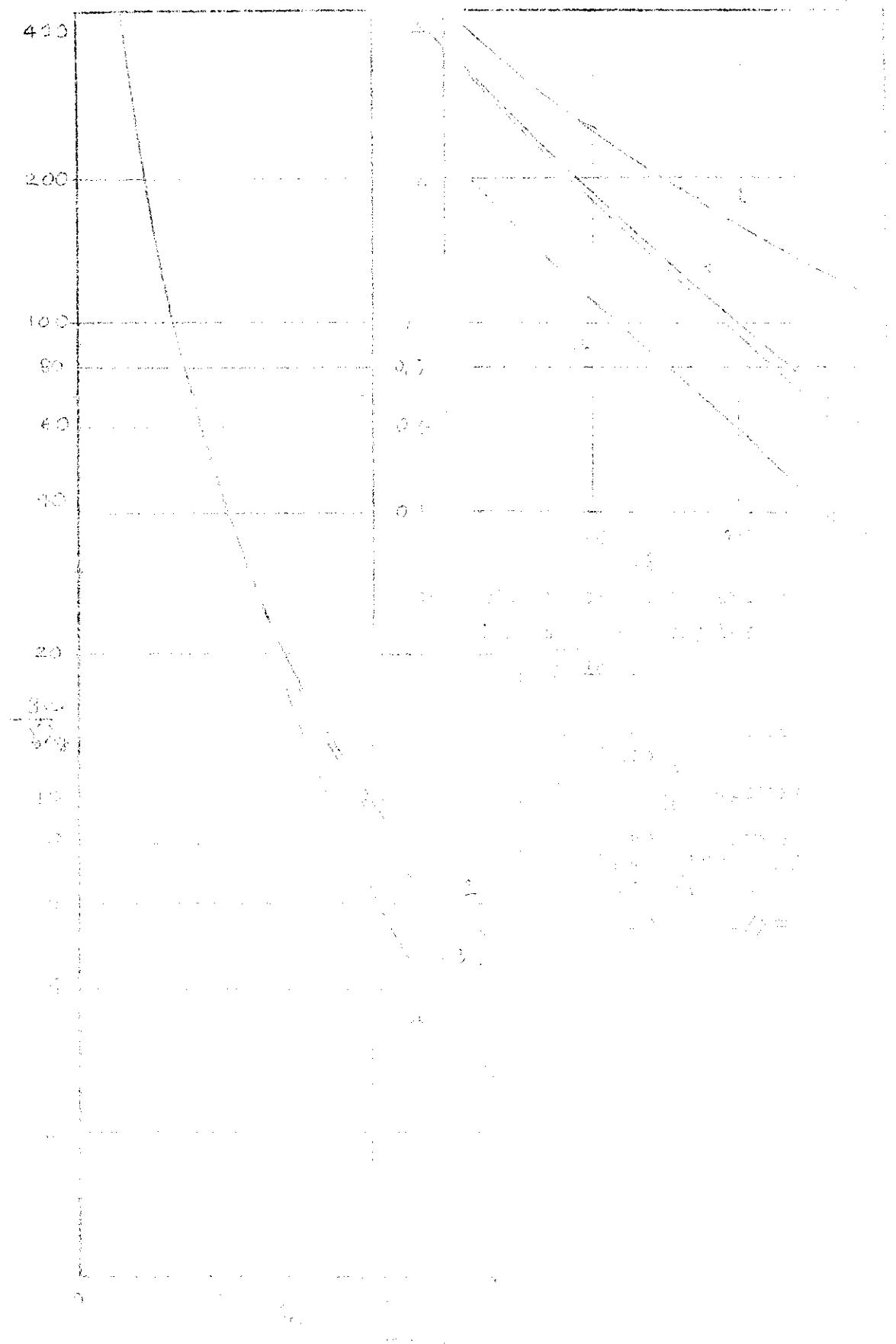
(1) Small value of Δ is $\frac{2d}{a}$,

(2) $\cot^2 \frac{\pi d}{2a}$ is much smaller,

(3) $E(\Delta) - \Delta F(\Delta)$ from curve in (108t)³³ of Δ vs Δ'

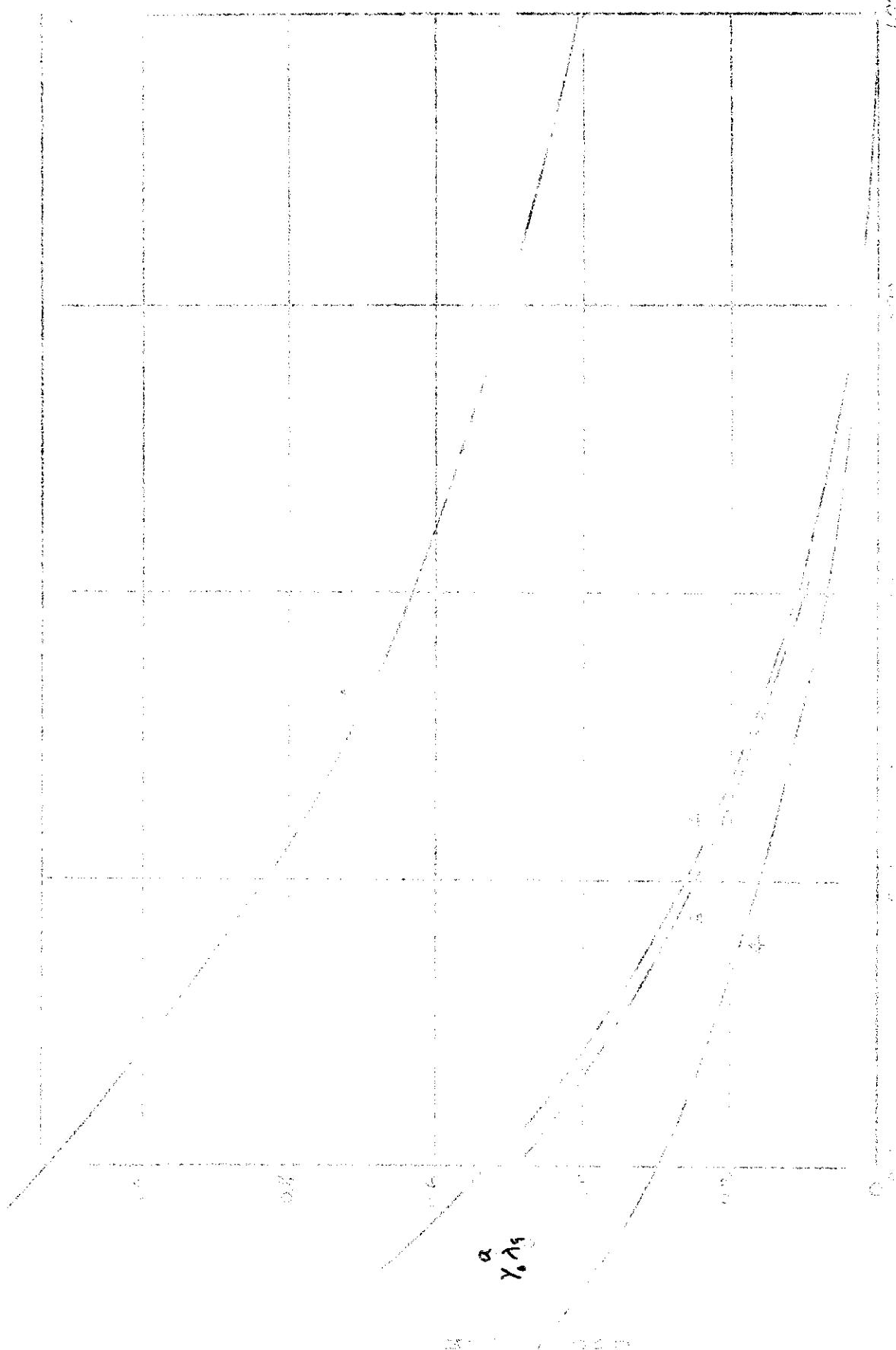
(4) Precise value from (108t) vs curve in (108n) of Δ vs Δ'

Surprisingly low surface resistance suspension of first as above
gives us just the right result, the "true" value reported by Leibinger.



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12. Energy Emitted from a Cavity through Window into Free Space.

Bethe's² equations (3) and (4) are not consistent with (1.3a)/(2.5) by a constant (27)². This makes his equation (1) give an energy emitted that is 46.6 times the correct value, i.e. assumptions are definitely not valid at the beginning of his paper as he claims.

My first step will therefore be to look at the derivation from number 35 and compare with mine. Here it is claimed that the losses are 10% which should be divided by 2 since they hold for one half of the light emitted considered. It would be to think that this is the case because of the assumption of $\epsilon^2 = \epsilon_0$ and $\mu^2 = \mu_0$ in the derivation. But this is not true. The losses are due to absorption in the metal and dielectric material. The losses are given by

$$\frac{S}{Q_C} = \frac{\omega^4}{12\pi^2} \int d\Omega \left(\epsilon^2 + \mu^2 \right)^{1/2} \frac{P}{P_0} \left(\frac{P}{P_0} - \frac{P}{P_{00}} \right)^2$$

$$(111a) \quad \frac{S}{Q_C} = \frac{\omega^4}{12\pi^2} \int d\Omega \left(\epsilon^2 + \mu^2 \right)^{1/2} \left(\frac{P}{P_0} - \frac{P}{P_{00}} \right)^2$$

$$(111b) \quad \frac{S}{Q_C} = \frac{\omega^4}{12\pi^2} \int d\Omega \left(\epsilon^2 + \mu^2 \right)^{1/2} \left(\frac{P}{P_0} - \frac{P}{P_{00}} \right)^2 \left(\frac{1}{\epsilon^2 + \mu^2} \right)$$

Now this is a very important result and it is the standard form of the formula for calculating the output of the cavity. In the case of a rectangular

$$Q_C = \frac{\omega^4}{12\pi^2} \int d\Omega \left(\epsilon^2 + \mu^2 \right)^{1/2} \left(\frac{P}{P_0} - \frac{P}{P_{00}} \right)^2 \left(\frac{1}{\epsilon^2 + \mu^2} \right)$$

$$(111a) \quad \frac{1}{Q_C} = \frac{\omega^4}{12\pi^2} \frac{\int d\Omega \left(\epsilon^2 + \mu^2 \right)^{1/2} \left(\frac{P}{P_0} - \frac{P}{P_{00}} \right)^2 \left(\frac{1}{\epsilon^2 + \mu^2} \right)}{\int d\Omega \left(\epsilon^2 + \mu^2 \right)}$$

$$(111b) \quad \frac{1}{Q_C} = \frac{4\omega^4}{12\pi^2} \frac{\epsilon \left(\frac{P}{P_0} \right) \epsilon_{air}^2 + \mu \left(\frac{M}{P_0} \right) M_0^2 + \left(\frac{P}{P_0} \right)^2 \left(\frac{1}{\epsilon^2} \right)}{\int \left(\epsilon^2 + \mu^2 \right) d\Omega}$$

Sample calculation on cavity power can be seen on p. 26 and 33.

from (111b)

$$\frac{1}{Q_c} = \frac{k^3}{12\pi} \frac{\left(\frac{M_1}{E}\right)^2 (H_0 l)^2}{U_E} = \frac{k^3 \nu \pi^2 h^2 E_0^2 J_1(k_0 d)}{12\pi \cdot 256 \eta^2 \pi^2 k_0 E_0^2 J_1(k_0 d)}$$

$$\frac{1}{Q_c} = \frac{16 \pi^2}{12,800} \frac{h d^4}{\alpha^2 \lambda^3} \quad Q_c = \frac{192}{\pi^2} \frac{\alpha^2 \lambda^3}{h d^4} \quad (111c)$$

for M_{100} compare with $\tau = 100$, $\nu = 10^6$, $h = 3.85$ cm.

$$Q_c = \frac{192}{\pi^2} \frac{675 \times 132^2}{\alpha^4 \lambda^3} \quad \frac{675}{\alpha^4 \lambda^3} \quad (111d)$$

13. Energy Emitted Through Window into Waveguide.

The normal modes in the waveguide are given by (98). The Poynting vector is:

$$S = \frac{1}{2} \int \vec{E} \times \vec{H}^* \cdot \hat{n} dS \quad (90)$$

$$S = \frac{1}{2} \left(\left(\sum_a A_a \vec{E}_a \cdot e^{-j\omega_a t} \right) \cdot \left(\sum_b A_b \vec{H}_b \cdot e^{+j\omega_b t} \right) \right) dS =$$

$$(112) \quad S = \frac{1}{2} \sum_a |A_a|^2 \left(\vec{E}_a \cdot \vec{H}_a \right) dS = \frac{1}{2} \sum_a |A_a|^2 S_a$$

$$\text{By (iv)(ii): } (113) \quad S = \sum_a \frac{\omega_a^2}{2Q_a} \left[M_a \left(\vec{H}_a \cdot \vec{H}_a \right) + N_a \left(\vec{E}_a \cdot \vec{E}_a \right) + j \left(\vec{P} \vec{E}_a \cdot \vec{H}_a \right) \right]$$

In general there can be intercoupling between various components of electric and magnetic moment of the window that cannot be written into free space.

For a complex writing field $\vec{H} = \vec{H}_0 e^{j\phi}$ where H_0 and ϕ are real,

$$(114) \quad S = \frac{\omega^2}{2Q_a} \left[M_a H_a^2 + N_a E_a^2 + jH_a E_a \right]$$

It would appear from the definition that such terms could be added simply to sum of the P in (113) to be carried out in (98).

Resonant Resonator Window

$H_{res} = H_0 e^{j\phi}$ is the window voltage component of (98).

$$(115)(a) \quad S = \frac{\omega^2}{2Q_a} \left[T \frac{H_0^2}{2} + H_0 \right] \quad (115)$$

For radiation loss $P_R = \frac{M_a H_a^2}{2Q_a}$ (115)

$$(115)(b) \quad \text{Radiation loss } P_R = \frac{M_a H_a^2}{2Q_a} = \frac{M_a^2}{2Q_a} \frac{H_0^2}{2} = \frac{M_a^2}{2Q_a} \frac{H_0^2}{2} \text{ (115)}$$

$$\frac{1}{Q_a} = \frac{\omega^2}{2Q_a} \frac{M_a^2}{H_0^2} \frac{H_0^2}{2} = \frac{\omega^2}{2Q_a} \frac{M_a^2}{H_0^2} \quad \text{as } H_0 \ll T \text{ (115)}$$

(115a)

$$(115a)(88) \quad \frac{1}{\zeta_0} = \frac{1}{2k} \frac{\lambda^2}{\sqrt{p^2 + k^2}} \frac{N^2 \pi^2 \ln L^2}{256 \pi^2 R^2 c^2} \left(\frac{E}{P}\right)^2$$

$$Q_C = \frac{\pi^2}{\lambda^2} \cdot \frac{E^{0.5} N^2}{8 \pi d^4} \quad (116)$$

For TH_{O_2} oscillator, $\Delta = 0.1^{\circ}$, $\alpha = 0.001$, $\beta = 0.0001$
 $\lambda = 3.35 \text{ mm. } d_{\text{eff}} = 0.93 \text{ mm. } (4^{\circ})$

$$Q_C = \frac{130}{\lambda^2} \cdot \frac{E^{0.5} N^2}{8 \pi d^4} \cdot \frac{(0.1)^2}{(0.001)^2} \cdot \frac{(0.0001)^2}{(0.93)^4} = 1.2 \times 10^{-10}$$

$$\begin{aligned} Q_C &= \frac{130}{(3.35)^2} \cdot \frac{E^{0.5} N^2}{8 \pi d^4} \cdot \frac{(0.1)^2}{(0.001)^2} \cdot \frac{(0.0001)^2}{(0.93)^4} \\ &\approx 1.2 \times 10^{-10} \cdot \frac{E^{0.5} N^2}{8 \pi d^4} \cdot \frac{10^{-4}}{10^{-6}} \cdot \frac{10^{-8}}{0.8 \times 10^{-8}} \\ &\approx 1.2 \times 10^{-10} \cdot \frac{E^{0.5} N^2}{8 \pi d^4} \cdot 12.5 \cdot 10^{-1} \\ &\approx 1.2 \times 10^{-10} \cdot \frac{E^{0.5} N^2}{8 \pi d^4} \cdot 1.25 \times 10^{-3} \end{aligned}$$



Report DIP 1000 - Objects and Definitions.

Object	Type	Definition	Value	Unit	Notes
1	Point	Point A	100	mm	
2	Point	Point B	100	mm	
3	Line	Line AB	100	mm	
4	Circle	Circle C	50	mm	
5	Circle	Circle D	50	mm	
6	Circle	Circle E	50	mm	
7	Circle	Circle F	50	mm	
8	Circle	Circle G	50	mm	
9	Circle	Circle H	50	mm	
10	Circle	Circle I	50	mm	
11	Circle	Circle J	50	mm	
12	Circle	Circle K	50	mm	
13	Circle	Circle L	50	mm	
14	Circle	Circle M	50	mm	
15	Circle	Circle N	50	mm	
16	Circle	Circle O	50	mm	
17	Circle	Circle P	50	mm	
18	Circle	Circle Q	50	mm	
19	Circle	Circle R	50	mm	
20	Circle	Circle S	50	mm	
21	Circle	Circle T	50	mm	
22	Circle	Circle U	50	mm	
23	Circle	Circle V	50	mm	
24	Circle	Circle W	50	mm	
25	Circle	Circle X	50	mm	
26	Circle	Circle Y	50	mm	
27	Circle	Circle Z	50	mm	
28	Circle	Circle AA	50	mm	
29	Circle	Circle BB	50	mm	
30	Circle	Circle CC	50	mm	
31	Circle	Circle DD	50	mm	
32	Circle	Circle EE	50	mm	
33	Circle	Circle FF	50	mm	
34	Circle	Circle GG	50	mm	
35	Circle	Circle HH	50	mm	
36	Circle	Circle II	50	mm	
37	Circle	Circle JJ	50	mm	
38	Circle	Circle KK	50	mm	
39	Circle	Circle LL	50	mm	
40	Circle	Circle MM	50	mm	
41	Circle	Circle NN	50	mm	
42	Circle	Circle OO	50	mm	
43	Circle	Circle PP	50	mm	
44	Circle	Circle QQ	50	mm	
45	Circle	Circle RR	50	mm	
46	Circle	Circle SS	50	mm	
47	Circle	Circle TT	50	mm	
48	Circle	Circle UU	50	mm	
49	Circle	Circle VV	50	mm	
50	Circle	Circle WW	50	mm	
51	Circle	Circle XX	50	mm	
52	Circle	Circle YY	50	mm	
53	Circle	Circle ZZ	50	mm	
54	Circle	Circle AAA	50	mm	
55	Circle	Circle BBB	50	mm	
56	Circle	Circle CCC	50	mm	
57	Circle	Circle DDD	50	mm	
58	Circle	Circle EEE	50	mm	
59	Circle	Circle FFF	50	mm	
60	Circle	Circle GGG	50	mm	
61	Circle	Circle HHH	50	mm	
62	Circle	Circle III	50	mm	
63	Circle	Circle JJJ	50	mm	
64	Circle	Circle KKK	50	mm	
65	Circle	Circle LLL	50	mm	
66	Circle	Circle MMM	50	mm	
67	Circle	Circle NNN	50	mm	
68	Circle	Circle OOO	50	mm	
69	Circle	Circle PPP	50	mm	
70	Circle	Circle QQQ	50	mm	
71	Circle	Circle RRR	50	mm	
72	Circle	Circle SSS	50	mm	
73	Circle	Circle TTT	50	mm	
74	Circle	Circle UUU	50	mm	
75	Circle	Circle VVV	50	mm	
76	Circle	Circle WWW	50	mm	
77	Circle	Circle XXX	50	mm	
78	Circle	Circle YYY	50	mm	
79	Circle	Circle ZZZ	50	mm	
80	Circle	Circle AAAA	50	mm	
81	Circle	Circle BBBB	50	mm	
82	Circle	Circle CCCC	50	mm	
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84	Circle	Circle EEEE	50	mm	
85	Circle	Circle FFFF	50	mm	
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260	Circle	Circle YLLL	50	mm	
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APPENDIX D
COMPARISON OF PREDICTED AND MEASURED.

TABLE D-1
Comparison of Predicted and Measured
Flow Velocities in the Main Channel

by

ASPHALTIC COATINGS AND POLYMER-BASED PAVEMENTS

Asphaltic coatings and polymer-based pavements are two types of materials used in the construction of roads. Asphaltic coatings are typically made from asphalt and aggregate, while polymer-based pavements are made from a polymer and aggregate. Both materials have different properties and are used for different purposes.

Asphaltic coatings are often used as a sealant or protective layer on existing roads. They are applied to the surface of the road to protect it from water damage and to prevent the loss of aggregate. Polymer-based pavements are often used as a base layer for new roads. They are applied to the ground to provide a smooth and stable surface for vehicles to travel on.

The main difference between asphaltic coatings and polymer-based pavements is the way they are applied. Asphaltic coatings are usually applied by hand, while polymer-based pavements are often applied by machine. This can affect the quality of the final product.

Another difference is the type of aggregate used. Asphaltic coatings typically use crushed stone or gravel as their aggregate, while polymer-based pavements often use sand or fine aggregate. This can also affect the quality of the final product.

Both asphaltic coatings and polymer-based pavements have their own advantages and disadvantages. Asphaltic coatings are relatively inexpensive and easy to apply, but they may not be as durable as polymer-based pavements. Polymer-based pavements are more expensive and require more time to apply, but they are more durable and can withstand harsh weather conditions.

In conclusion, asphaltic coatings and polymer-based pavements are two different types of materials used in the construction of roads. They both have their own unique properties and are used for different purposes. The choice of which material to use depends on the specific needs of the project.

Overall, asphaltic coatings and polymer-based pavements are important components of modern infrastructure. They help to keep our roads safe and functional for years to come.

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APPENDIX D - METHOD OF FINDING THICKNESS
OF RESONATOR

For small width holes the effect of the finite thickness of the resonator wall becomes important. Curves are available for the change in field over emitted in waveguide in terms of the hole diameter relative equivalent network in the waveguide (Kazalik, 1957) or in terms of current density, since the corresponding expression is readily available for rectangular holes, a computation is made as follows:

$$\frac{E_x}{E_{x0}} = \frac{\pi^2}{\pi^2 + 4} \left(\frac{d}{w} \right)^2 \quad \text{for small thickness}$$

To obtain the thickness of the resonator wall required to reduce the field in the waveguide by a factor of 1.05, we have

$$\frac{E_x}{E_{x0}} = \frac{\pi^2}{\pi^2 + 4} \left(\frac{d}{w} \right)^2 = 0.95$$

$$\left(\frac{d}{w} \right)^2 = \frac{4}{\pi^2} \cdot \frac{1}{0.95} = 1.27$$

$$d = w \sqrt{1.27} = 1.12w$$

for $d \ll w$

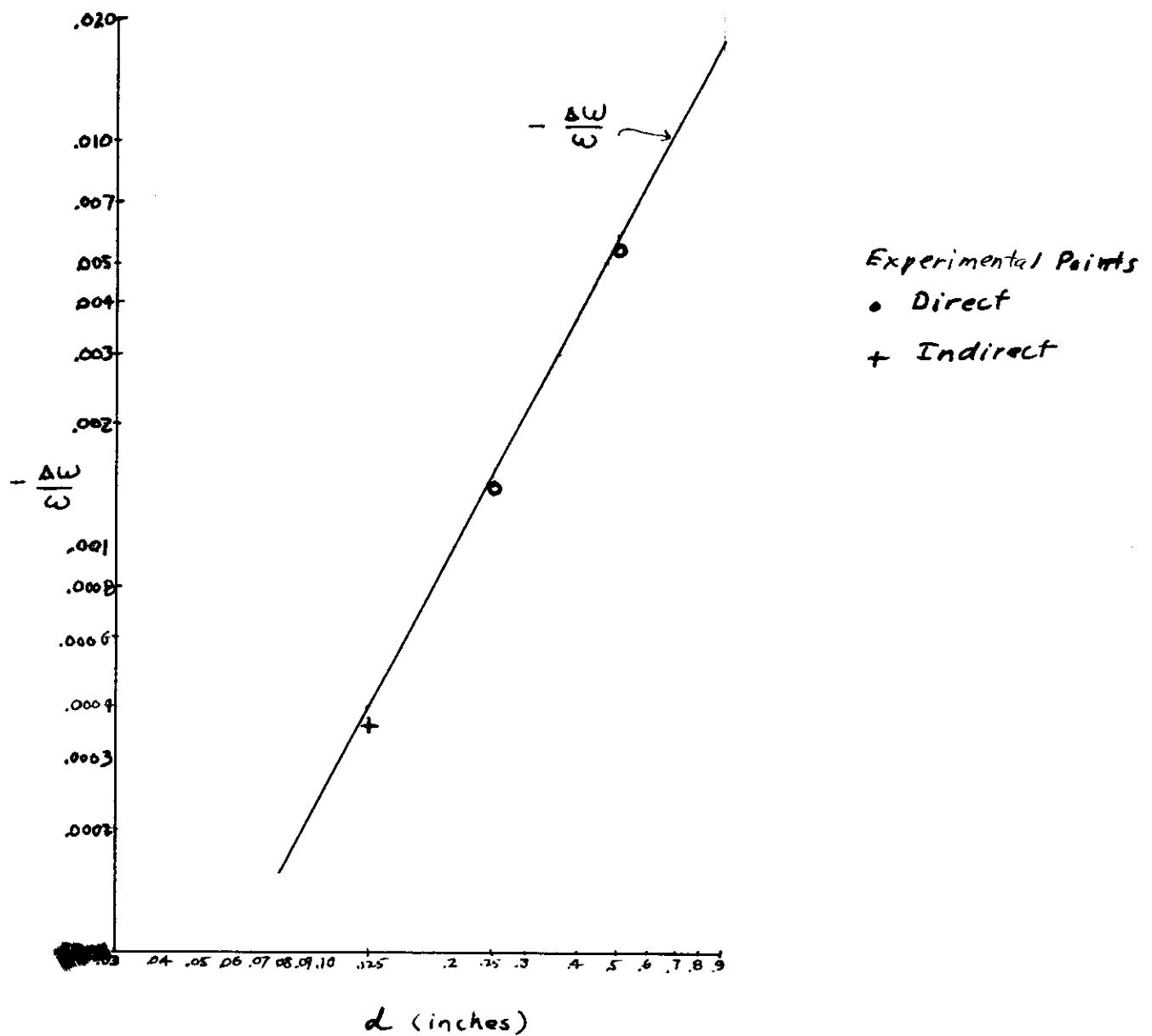
For the case of a rectangular hole of width w and height h the field reduction applied to the waveguide is given by the following:

$\frac{w}{h}$	$\frac{d}{w}$	$\frac{d}{h}$	$\frac{d}{w+h}$	E_x/E_{x0}	2^{nd} d	E_{x0}/E_x	$\frac{d}{w}$
0.951	2.07	0.97	1.25	0.957	1.14	3.46	2.06
0.943	2.16	1.0	1.28	0.95	0.992	1.77	2.16
0.920	2.27	1.07	1.36	0.946	0.982	1.997	2.27

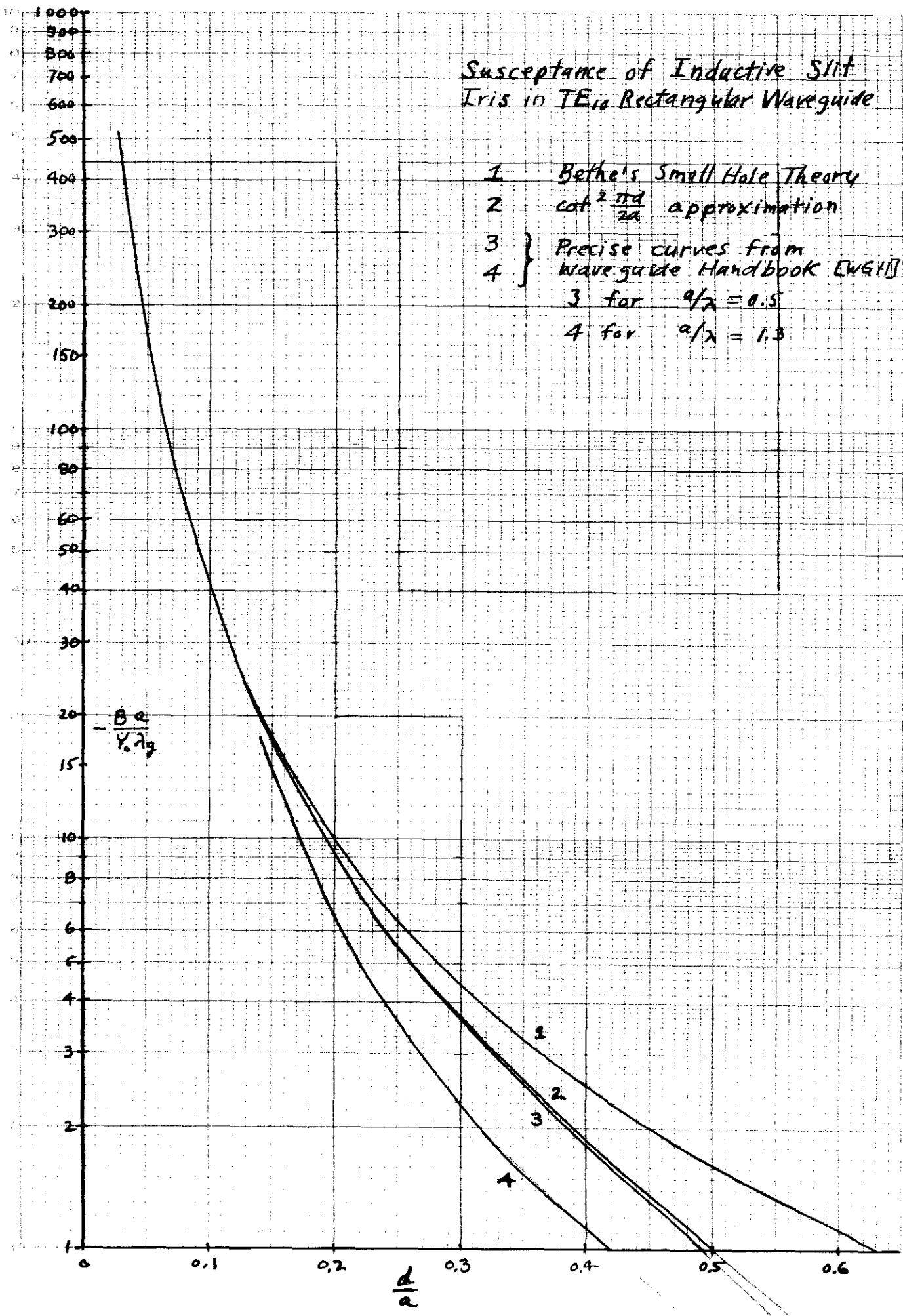
- 1a R. A. Bethe, "Theory of scattering by small holes," Proc. Roy. Soc. (London) Series A, Vol. 173, No. 95, Jan 23, 1935.
- 1b ...A. Bethe (declassified in use of ref 1a), Phys. Rev., Vol. 46, p. 165 (1934).
- 2 A. A. Bethe, "Theory of Compton Scattering and Unites," Proc. Roy. Soc. (London) Series A, Vol. 173, No. 95, Jan 23, 1935.
- 3 G. J.尊重 (S. S. Wilson), "Compton Scattering by Small Holes," Annals of Phys. (N.Y.), Vol. 1, No. 1, p. 13 (1953).
- 4 Bethe, et al. (Ref. 2).
- 5 J. D. Bjorken and S. Drell, "Relativistic Quantum Mechanics; Relativistic Corrections to Compton Scattering," p. 103.
- 6 G. E. Brown, "Relativistic Compton Scattering," p. 105.
- 7 G. E. Brown, "Relativistic Compton Scattering," p. 105. Reprinted from Ref. 6.
- 8 G. E. Brown, "Relativistic Compton Scattering," p. 105. Reprinted from Ref. 6.
- 9 G. E. Brown, "Relativistic Compton Scattering," p. 105. Reprinted from Ref. 6.
- 10 G. E. Brown, "Relativistic Compton Scattering," p. 105. Reprinted from Ref. 6.
- 11 G. E. Brown, "Relativistic Compton Scattering," p. 105. Reprinted from Ref. 6.
- 12 G. E. Brown, "Relativistic Compton Scattering," p. 105. Reprinted from Ref. 6.

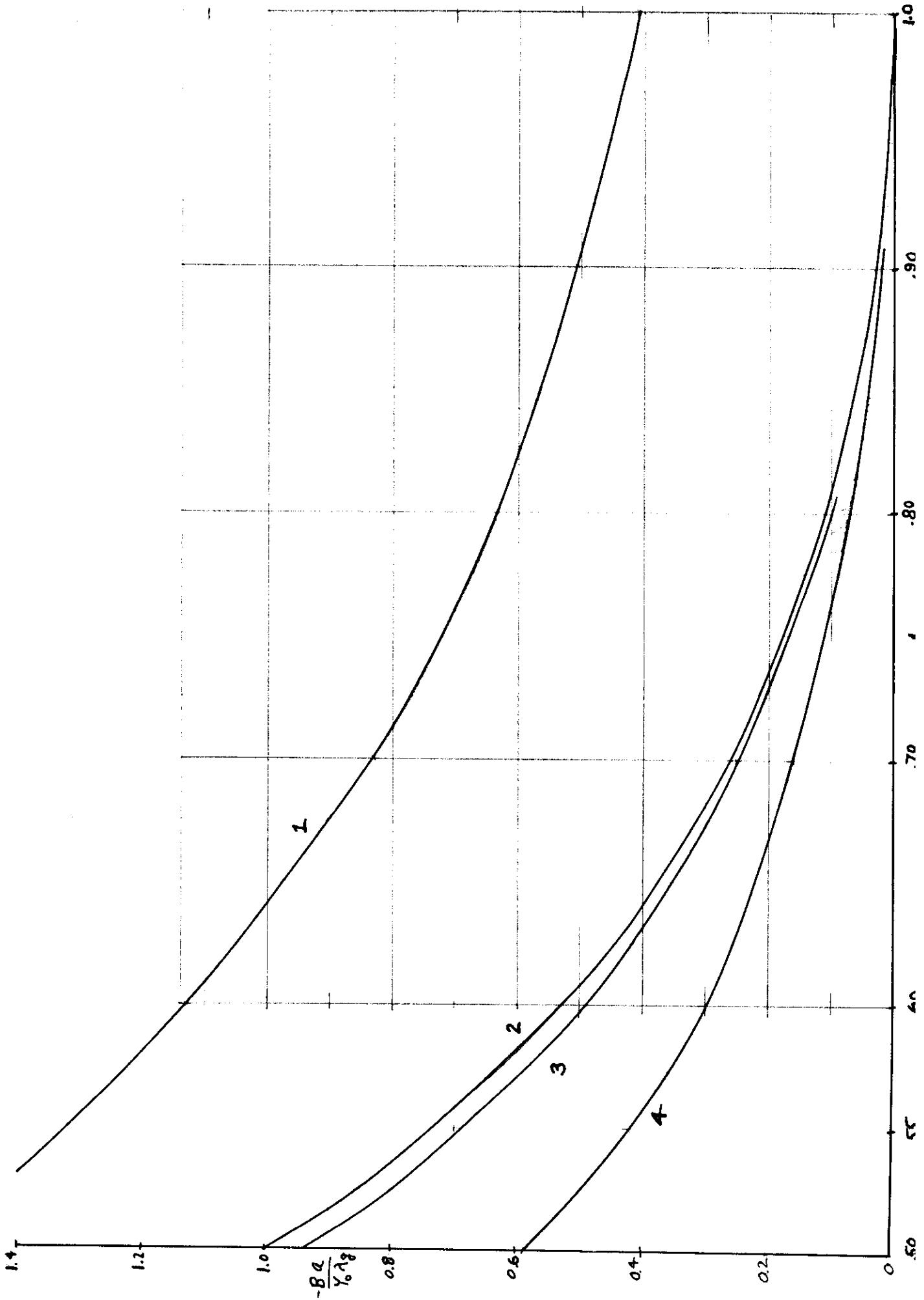
Graph of
 $\frac{\Delta\omega}{\omega}$ vs. d

Frequency Shift in TM₀₂₀ Cylindrical Resonator Due To Inductive Slit Iris.



Susceptance of Inductive Slit
Iris in TE₁₀ Rectangular Waveguide





Comparison of Approximate Theoretical Results with Experiment
 o Direct + D. G. J. M. Measurement

